RADIATION SPECTRUM OF THE SYSTEM OF ELECTRONS MOVING IN A SPIRAL IN TRANSPARENT MEDIUM

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The spectral distributions of the radiation power for the system of non interacting electrons moving in magnetic fields in an isotropic transparent medium are studied by combining analytical and high-accuracy numerical methods. Special attention is given to the research of the fine structure of the synchrotron radiation spectral distribution of the radiation power of one, two, and three electrons moving in a spiral in a transparent medium with relativistic transversal component (the component perpendicular to the magnetic field) of velocity. Using the expressions for spectral distribution of the average radiation power of three electrons moving one by one in a spiral in transparent isotropic medium the synchrotron radiation spectrum in dependence of their location along a spiral for the fist time is investigated. The influence of the coherence factor on the spectrum of synchrotron radiation for two and three electrons is studied.

Key words: charge transfer, coherent electromagnetic radiation, synchrotron radiation, current density, photoelectric emission, Lorentz's self-interaction method.

1. INTRODUCTION

In 1898 Lienard [1] gave a formula for the rate of radiation from a centripetally accelerated charge. In 1908 Schott [2] further developed the classical radiation theory of charge moving in a circle in connection with the study of atomic models. The classical expressions [3–4] describing the spectrum emitted by high-energy electrons moving in circular orbits received experimental verification. In the exploration by Elder, Langmuir and Pollock [5], radiometric measurements were carried out in the visible and near ultraviolet regions. Such investigations in the visible region are presented in Ref. [6]. The experimental results presented in [5, 6] are in satisfactory agreement to the theories by Ivanenko and Sokolov [3] and Schwinger [4]. The experimental investigation in the ultraviolet [7–8] and in X-ray regions [9] is also in concordance with the classical radiation theory [3–4].

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The properties of synchrotron radiation of charged particles moving in a circle or spiral in vacuum are studied in papers [10–20]. In paper [11] to examine the generation of coherent electromagnetic radiation from a system of non-interacting charges moving spirally in a constant uniform magnetic field in vacuum. The particularity of diverse property of synchrotron radiation of charges moving in vacuum in magnetic field is examined by Ternov in report [12].

The electromagnetic radiation spectrum of one electron moving in a medium in magnetic field is investigated in papers [13–15, 18–19, 21–26]. The synchrotron spectrum of two electrons moving in a spiral in medium combining the analytical transformations and numerical calculations is investigated in papers [25–26].

The aim of this paper is using the improved the Lorentz's self-interaction method, to investigate the radiation spectrum of electrons moving in a spiral in magnetic field in transparent media. Using the exact integral relationships for the spectral distribution of radiation power, the fine structure of the synchrotron radiation spectrum of three electrons moving in a spiral in a medium are obtained for the first time. The Doppler effect influence on particularities of the radiation spectrum of the one, two, and three electrons moving in a spiral in medium is investigated.

2. SPECTRAL DISTRIBUTION OF RADIATION POWER OF SYSTEM OF ELECTRONS MOVING ALONG A SPIRAL IN TRANSPARENT MEDIUM

By means of the improved Lorentz's self-interaction method we will obtain the principal relationships which will be used in our analytical transformations and numerical calculations for the radiation spectrum of the system of electrons moving in a spiral in magnetic field in a transparent isotropic medium. The time-averaged radiation power $\overline{P}^{rad}$ of the system of electrons moving in magnetic field is expressed in [27] as

$$\overline{P}^{rad} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left\{ \int_{\tau} \left(\vec{j}\left(\vec{r},t\right) \frac{1}{c} \frac{\partial \vec{A}^{Dir}\left(\vec{r},t\right)}{\partial t} - \rho\left(\vec{r},t\right) \frac{\partial \varphi^{Dir}\left(\vec{r},t\right)}{\partial t} \right) d\vec{r} \right\} dt. \qquad (1)$$

Here $\vec{j}\left(\vec{r},t\right)$ is the current density and $\rho\left(\vec{r},t\right)$ is the charge density. The integration is over some volume τ. According to the hypothesis of Dirac [28], the scalar $\varphi^{Dir}\left(\vec{r},t\right)$ and vector $\vec{A}^{Dir}\left(\vec{r},t\right)$ potentials are defined as a half-difference of the retarded and advanced potentials:

$$\varphi^{Dir} = \frac{1}{2}\left(\varphi^{ret} - \varphi^{adv}\right), \qquad \vec{A}^{Dir} = \frac{1}{2}\left(\vec{A}^{ret} - \vec{A}^{adv}\right). \qquad (2)$$

Then according to [27], the source functions of the system of N electrons are defined as

$$\vec{j}(\vec{r},t)=\sum_{l=1}^{N}\vec{V}_l(t)\rho_l(\vec{r},t), \quad \rho(\vec{r},t)=\sum_{l=1}^{N}\rho_l(\vec{r},t),$$
$$\rho_l(\vec{r},t)=e\delta(\vec{r}-\vec{r}_l(t)), \tag{3}$$

where $\vec{r}_l(t)$ and $\vec{V}_l(t)$ are the motion law and the velocity of the l^{th} electrons, respectively.

We study the system of N electrons moving one by one in a spiral in transparent media. The law of motion and the velocity of the l^{th} electron are given by the expressions

$$\vec{r}_l(t)=r_0\cos\{\omega_0(t+\Delta t_l)\}\,\vec{i}+r_0\sin\{\omega_0(t+\Delta t_l)\}\,\vec{j}+V_{\parallel}(t+\Delta t_l)\,\vec{k},$$
$$\vec{V}_l(t)=\frac{d\vec{r}_l(t)}{dt}. \tag{4}$$

Here $r_0=V_{\perp}\omega_0^{-1}$, $\omega_0=ceB^{ext}\tilde{E}^{-1}$, $\tilde{E}=c\sqrt{p^2+m_0^2c^2}$, the magnetic induction vector $\vec{B}^{ext}\parallel 0Z$, $V_{\perp}$ and $V_{\parallel}$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass.

The time-averaged radiation power of the system of electrons we obtain after substituting expressions (2)–(4) into (1). Then

$$\overline{P}^{rad}=\int_0^{\infty}W(\omega)d\omega, \tag{5}$$

$$W(\omega)=\frac{2e^2}{\pi c^2}\int_0^{\infty}dx\mu(\omega)\omega S_N(\omega)\frac{\sin\left\{\frac{n(\omega)}{c}\omega\eta(x)\right\}}{\eta(x)}\times$$
$$\times\cos\omega x\left[V_{\perp}^2\cos(\omega_0 x)+V_{\parallel}^2-\frac{c^2}{n^2(\omega)}\right], \tag{6}$$

where $\eta(x)=\sqrt{V_{\parallel}^2x^2+4\frac{V_{\perp}^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$, $\mu(\omega)$ is the magnetic permeability, $n(\omega)$ is the refraction index, ω is the cyclic frequency, and c is the velocity of light in vacuum.

In the case of N electrons moving one by one along a spiral the coherence factor $S_N(\omega)$ takes the form [27]:

$$S_N(\omega) = \sum_{l,j=1}^{N} \cos\left\{\omega\left(\Delta t_l - \Delta t_j\right)\right\}. \tag{7}$$

Starting from relationships (5) and (6) at $V_{\parallel} < c/n(\omega)$ the contribution of separate harmonics to the averaged radiation power can be written as

$$\begin{aligned} \overline{P}^{rad} = \frac{e^2}{c^3}\sum_{m=1}^{\infty}\int_0^{\infty} d\omega\mu(\omega)n(\omega)S_N(\omega)\omega^2 \times \\ \times\int_0^{\pi}\sin\theta d\theta\delta\left\{\omega\left(1-\frac{n(\omega)}{c}V_{\parallel}\cos\theta\right)-m\omega_0\right\}\times \\ \times\left\{V_{\perp}^2\left[\frac{m^2}{q^2}J_m^2(q)+J_m'^2(q)\right]+\left(V_{\parallel}^2-\frac{c^2}{n^2(\omega)}\right)J_m^2(q)\right\}. \end{aligned} \tag{8}$$

where $q = \dfrac{n(\omega)}{c}\dfrac{\omega}{\omega_0}V_{\perp}\sin\theta$, $J_m(q)$ and $J'_m(q)$ are the Bessel function with integer index and its derivative, respectively.

From relationship (8) one can conclude that each harmonic is a set of frequencies, which are determined from the solution of the equation

$$\omega\left(1-\frac{n(\omega)}{c}V_{\parallel}\cos\theta\right)-m\omega_0 = 0. \tag{9}$$

When ε and μ are constant, the boundaries of the m^{th} harmonic for $V_{\parallel} < c/n$ are given by the frequencies

$$\omega_m^{\max} = m\omega_0\left(1-\frac{nV_{\parallel}}{c}\right)^{-1}, \qquad \omega_m^{\min} = m\omega_0\left(1+\frac{nV_{\parallel}}{c}\right)^{-1}. \tag{10}$$

The coherence factor $S_1(\omega)$ of a single electron is defined as

$$S_N(\omega) = S_1(\omega) = 1. \tag{11}$$

In the case of two electrons the coherence factor $S_2(\omega)$ is defined as

$$S_2(\omega) = 2 + 2\cos(\omega\Delta t_{12}). \tag{12}$$

Here $\Delta t_{12} = \Delta t_2 - \Delta t_1$ is the time shift between the first and second electrons moving along a spiral. The analogous expression for the coherence factor was obtained by Bolotovskii [29].

The coherence factor $S_3(\omega)$ of three electrons takes the form

$$S_3(\omega) = 3 + 2\cos(\omega\Delta t_{12}) + 2\cos(\omega\Delta t_{23}) + 2\cos\{\omega(\Delta t_{12} + \Delta t_{23})\} \quad (13)$$

Here Δt_{23} is the time shift between the second and third electrons.

The total power emitted by a single electron [22, 24] (ε and μ are constant) is:

$$P_{med}^{tot} = \frac{2}{3}\frac{e^2 \mu n}{c^3}\omega_0^2 V_\perp^2 \left(1 - \frac{n^2 V^2}{c^2}\right)^{-2}, \quad (14)$$

3. FINE STRUCTURE OF THE RADIATION SPECTRUM OF ONE, TWO AND THREE ELECTRONS MOVING ALONG A SPIRAL IN MEDIUM

It is interesting to compare the radiation power spectral distribution for a single electron (curve 1 in Fig. 1) to those for two and three electrons (curve 2 in Fig. 2 and curve 3 in Fig. 3, respectively). Our numerical calculation of the

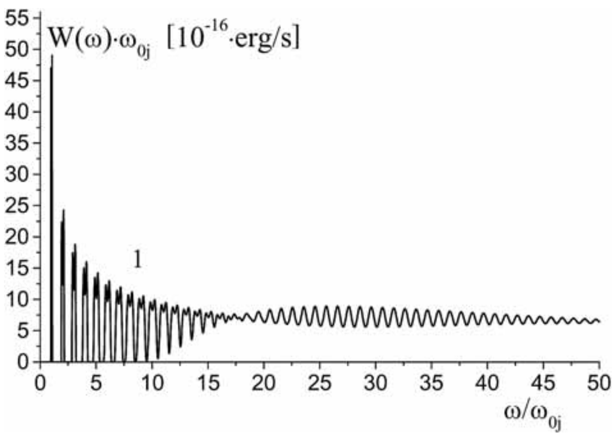


Fig. 1 – Synchrotron radiation spectrum of a single electron moving in a spiral in medium with radiation power $P_{med1}^{\text{int}} = 0.3293 \cdot 10^{-13}$ erg/s.

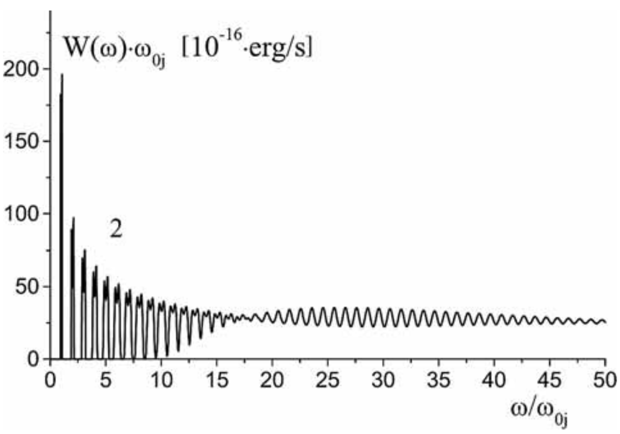


Fig. 2 – Synchrotron radiation spectrum of two electrons moving one by one in a spiral in medium for the time shift $\Delta t_{12}^{(2)} = 0.001 \cdot \pi / \omega_{02}$ with radiation power $P_{med2}^{\text{int}} = 0.1314 \cdot 10^{-12}$ erg/s.

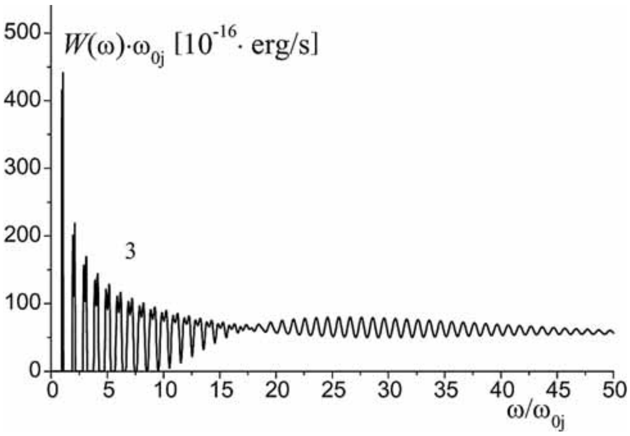


Fig. 3 – Synchrotron radiation spectrum of three electrons moving in a spiral in medium for the time shifts $\Delta t_{12}^{(3)} = \Delta t_{23}^{(3)} = 0.001 \cdot \pi / \omega_{03}$ with radiation power $P_{med3}^{\text{int}} = 0.2947 \cdot 10^{-12}$ erg/s.

radiation spectra were carried out on the basis of equations (5) and (6) taking into account the corresponding coherence factors. The spectral distribution of synchrotron radiation power was obtained for $B^{ext} = 1$ Gs, $n = 1.25$, $V_{\perp vac} = 0.231 \cdot 10^{11}$ cm/s, $V_{\| vac} = 0.15 \cdot 10^{10}$ cm/s, $c = 0.2997925 \cdot 10^{11}$ cm/s, $r_{0j} = 2067$ cm, $\omega_{0j} = 0.1118 \cdot 10^{8}$ rad/s (j = 1, 2, …, 5).

The total radiation power of a single electron in vacuum $P_{med1}^{tot} = 0.1016 \cdot 10^{-12}$ erg/s is calculated according to relationship (14). The radiation power $P_{med1}^{\text{int}} = 0.3293 \cdot 10^{-13}$ erg/s is determined after integration of relationships (5) and (6) at $S_N(\omega) = S_1(\omega) = 1$.

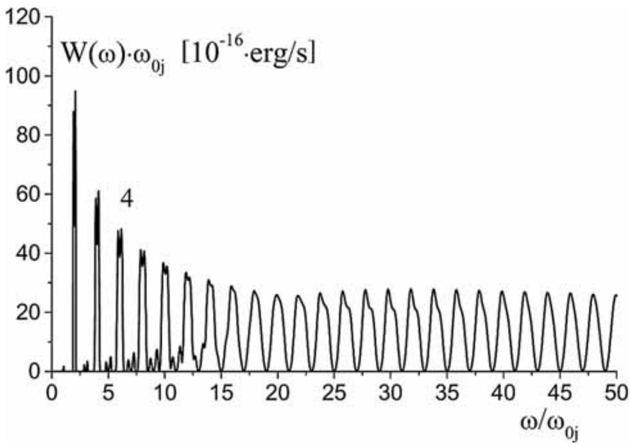
For the time shifts $\Delta t_{12}^{(2)} = 0.001 \cdot \pi / \omega_{02}$ the coherence factor $S_2(\omega) \cong 4$ and two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$ (curve 2 in Fig. 2), *i.e.* by a factor of four higher than a single electron ($P_{med2}^{\text{int}} \cong 4 \cdot P_{med1}^{\text{int}} = 0.1317 \cdot 10^{-12}$ erg/s). Here the upper index (2) and other similar ones further mark the corresponding curves in the plots.

For the time shifts $\Delta t_{12}^{(3)} = \Delta t_{23}^{(3)} = 0.001 \cdot \pi / \omega_{03}$ the coherence factor $S_3(\omega) \cong 9$ and three electrons radiate as a charged particle with the charge $3e$ and the rest mass $3m_0$ (curve 3 in Fig. 3), *i.e.* by a factor of nine higher than a single electron ($P_{med3}^{\text{int}} \cong 9 \cdot P_{med1}^{\text{int}} = 0.2964 \cdot 10^{-12}$ erg/s).

In the case of uniform location of two electrons (one the opposite sides of a convolution) at the time shifts $\Delta t_{12}^{4} = \pi / \omega_{04}$ any radiation at the frequencies $(2i-1)\omega_{04}$, i = 1, 2, …, 25 is absent (curve 4 in Fig. 4).

In the case of uniform location of three electrons along spiral at the time shifts $\Delta t_{12}^{(6)} = \Delta t_{23}^{(6)} = 2\pi/(3 \cdot \omega_{03})$ we have found that any radiation also is absent at the frequencies $(3i-2)\omega_{05}$ and $(3i-1)\omega_{05}$ (i = 1, 2, …, 17) (curve 5 in Fig. 5).

Fig. 4 – Spectral distribution of synchrotron radiation power of two electrons moving in a spiral in medium for $\Delta t_{12}^{(4)} = \pi / \omega_{04}$ and $P_{med4}^{\text{int}} = 0.6565 \cdot 10^{-13}$ erg/s.



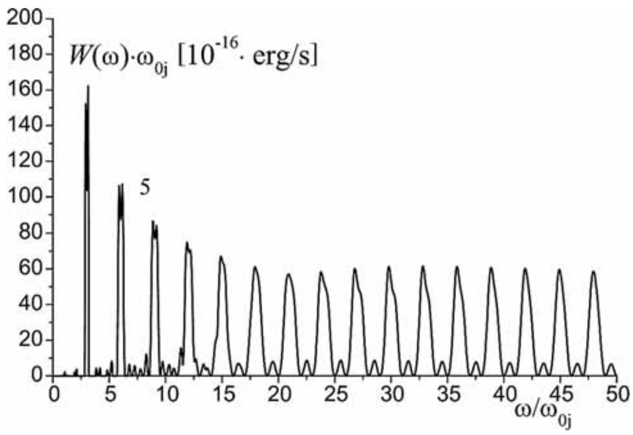


Fig. 5 – Spectral distribution of synchrotron radiation power of three electrons moving in a spiral in medium for the time shifts $\Delta t_{12}^{(5)} = \Delta t_{23}^{(5)} = 2\pi/(3 \cdot \omega_{05})$ and $P_{med5}^{\text{int}} = 0.9712 \cdot 10^{-13}$ erg/s.

The coherence factor leads to essential changes in the radiation power spectral distribution of two and three electrons in dependence on their position in a spiral (see Figs. 2 to 5).

The Doppler effect determines the band's boundaries of separate harmonics in the radiation spectra of one, two, and three electrons moving in spiral in a medium (see Figs. 1 to 5).

4. CONCLUSIONS

For small time shifts two electrons in medium radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, *i.e.* by a factor of four higher than a single electron.

At the uniform location of two electrons along a spiral at the time shifts $\Delta t_{12}^{(4)} = \pi / \omega_{04}$ between them the radiation at the frequencies $(2i-1)\,\omega_{04}$ (i = 1, 2, …, 25) is absent.

For small time shifts three electrons in a medium radiate as a charged particle with the charge $3e$ and the rest mass $3m_0$, *i.e.* by a factor of nine higher than a single electron.

For uniform location of three electrons along a spiral at the time shifts $\Delta t_{12}^{(5)} = \Delta t_{23}^{(5)} = 2\pi/(3\cdot\omega_{05})$ between them the radiation at the frequencies $(3i-2)\,\omega_{05}$ and $(3i-1)\,\omega_{05}$ (i = 1, 2, …, 17) is absent.

The influence of the Doppler effect determines the band's boundaries of the separate harmonics in the radiation spectra of the one, two, and three electrons in a medium.

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