GRAPHICAL METHOD OF BREAKING SURFACE DETERMINATION IN AN EARTH MASSIF BY USING THE CONDITION OF ULTIMATE EQUILIBRIUM*

DAN ZAROJANU

University of Suceava, Universităţii str. no. 13, RO 720225, Suceava, Romania
zarojanu@yahoo.com, zarojanu@eed.usv.ro,
Tel. +40-230-521664, +40-745615191, Fax: 0040-230-521664,

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It is known the fact that the ultimate equilibrium in a point from an earth massif can be graphically plotted, by the tangency condition of the characteristic line of that earth to the Mohr circle, corresponding to that effort.

The method consists in finding out the points locations and the directions of the planes of ultimate equilibrium. The curve of these directions will be the presumptive surface of breaking.

For the beginning, it considers a load with a focused force and in the end it shall make the suitable generalization.

1. GENERAL

The ultimate equilibrium in a point from an earth massif can be graphically plotted by the tangency condition of the characteristic line of that earth (Coulomb line) to the Mohr circle, corresponding to that effort.

Considering the existence of a plane state of tensions, the graphical representation of the ultimate equilibrium is that from Fig. 1, where σ_1, σ_2 – principal normal unit stress, c – specific cohesion and φ – angle of internal friction.

The method consists in finding out several points locations and the directions of the planes of its ultimate equilibrium. Then will be obtained the curve of these directions, by a certain method. This curve will describe the presumptive surface of breaking. For the beginning, it considers a certain load as a focused force. Then it shall make the suitable generalization, even not in this paper.

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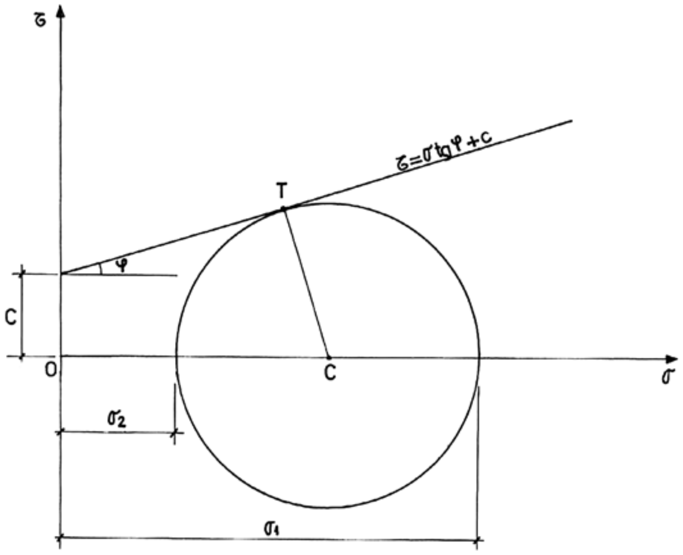

Fig. 1 – Graphical representation of the ultimate equilibrium for a point in an earth massif, in the hypothesis of a plane state of tensions.

2. FINDING OUT THE POINTS LOCATIONS AT ULTIMATE EQUILIBRIUM AND THE DIRECTIONS OF THE PLANES OF ULTIMATE EQUILIBRIUM CROSSING THROUGH THESE POINTS

The determination starts from the point situated in an ultimate equilibrium, situated on the vertical line of the application point of the focused force (see Fig. 2). We calculate the units stress σ_x, σ_z by using the Boussinesq equations. Then we calculate the normal principal units stress σ_1, σ_2 in terms of these, so as we could draw the Mohr circle.

$$\sigma_x = \frac{P(1-2\mu)}{2\pi}\left[\frac{1}{R(R+z)} - \frac{(2R+z)x^2}{(R+z)^2 R^3} - \frac{z}{R^3}\right] \tag{1}$$

$$\sigma_z = \frac{3P}{2\pi}\cdot\frac{z^3}{R^5}; \tag{2}$$

$$\tau_{zx} = \frac{3P}{2\pi}\cdot\frac{xz^2}{R^5}, \tag{3}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\sigma_x - \sigma_z\right)^2 + 4\tau_{xz}^2}, \tag{4}$$

where: μ – Poisson’s factor;

R – the positions vectors of the $M(x, z)$ points.

Because the problem was reduced to a plane one, then:

$$R = \sqrt{x^2 + z^2}; \tag{5}$$

P – known focused force.

From the tangency condition of the characteristic (Coulomb) line $(\tau = \sigma \cdot tg\varphi + c)$ at the Mohr circle, as well as Sokolovschi ultimate equilibrium condition, there are determined the positions of the points $M(x, z)$.

Sokolovschi condition:

$$\sin\varphi = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2 + \dfrac{2c}{tg\varphi}} \tag{6}$$

For this first point, where was reached the ultimate equilibrium, it will be $M_0(x_0, z_0)$, where $x_0 = 0$ and $z_0 = R$. Thus, this point is situated on the vertical line of the application point of the focused force

Knowing the main directions (1) and (2) and knowing that the plane of the ultimate equilibrium crossing through these points makes a $\frac{\pi}{4} + \frac{\varphi}{2}$ angle with the principal direction (1), it could draw the direction of the plane of ultimate equilibrium through such a point (see Fig. 2).

For the first point, the principal directions will be even the directions of *ox* and *oz* axles.

So:

$$\sigma_z = \sigma_1 = \frac{3P}{2\pi \cdot z_0^2} \Rightarrow z_0 \tag{7}$$

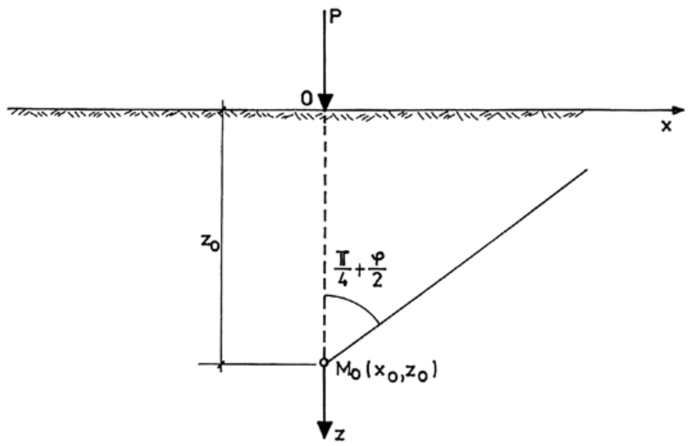

Fig. 2 – Finding out the point location at ultimate equilibrium and the direction of the planes of ultimate equilibrium cross through this point – situated on the vertical line of the application point of the focused force.

The following phase consists in finding out the position and direction of the plane ultimate equilibrium of a neighbouring point $M_1(x_1, y_1)$ that is in an ultimate equilibrium at the distance $x_1 = x \neq 0$ of M_0 (see Fig. 3), proceeding as above. In this case, by following the same way, it will be found z_1 and the direction of the plane ultimate equilibrium. M_1 will verify the characteristic line equation simultaneously with Mohr circle equation:

$$\frac{\sigma_1 - \sigma_2}{2}\cos\varphi = \left[\sigma_2 + \frac{\sigma_1 - \sigma_2}{2}(1 - \sin\varphi)\right] tg\varphi + c,$$

where: $\sigma_{1,2} = f(x, z)$, see (1), (2), (3), (4) and (5); c, φ are the internal characteristics of the earth.

They are known (see above).

$\Rightarrow z_1 = f(P, x, \mu, \varphi, c)$

Only x is unknown in the equation above, so that it could obtain the variation of z for several values of x parameter.

The same procedure will be used for another point M_2, then M_3 etc.

The last phase consists in drawing the curve of the directions for the planes of ultimate equilibrium (see Fig. 4), representing the breaking surface.

In the future, the method development and generalization, could consist in:

- Considerations regarding the possibility of finding a primitive that should describe more rigorously the breaking surface:
- Construction of a curve family by changing the value of the focused force and/or for other types of loads;

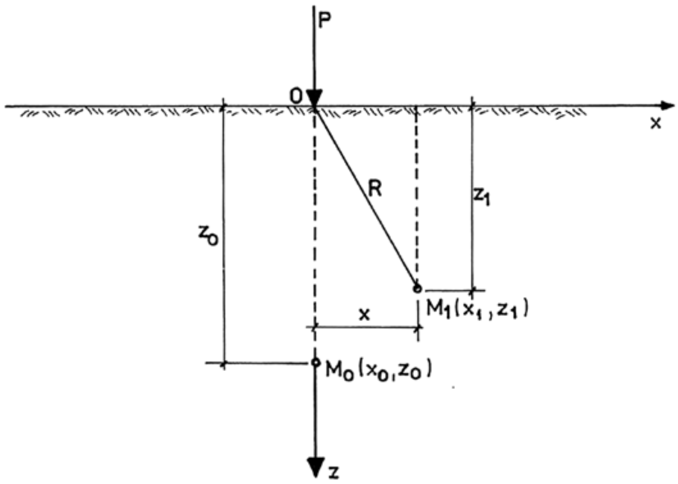

Fig. 3 – Finding out the position of a neighbouring point at ultimate equilibrium.

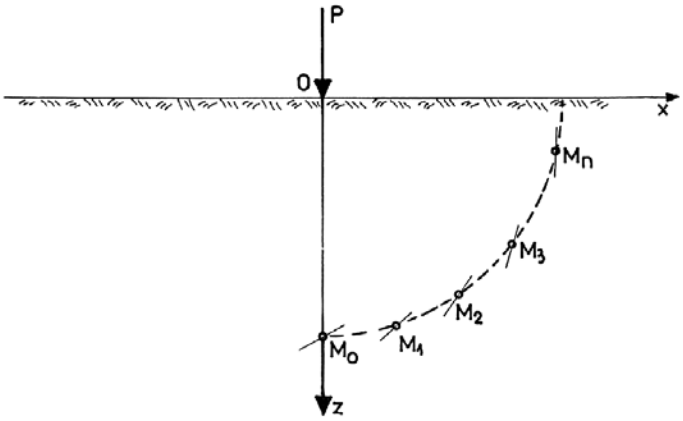

Fig. 4 – Construction of the breaking surface.

- Considerations regarding the accomplishment a nomogram or a table with several inputs that should create the possibility of generalization concerning the taking into consideration of several values of testing, and more earth types, respectively.

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