

ELASTIC SCATTERING BY A POROUS INCLUSION IN ELASTIC MATRIX*

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Received September 26, 2006

The goal of this paper is to study the scattering of acoustic waves on an air-filled porous cylinder in an elastic matrix. A comparison between the forward and backward far-field scattering give us the possibility to use the Foldy's theory in order to replace the porous medium by an "effective" elastic one. This last one are to be used to the determination of mechanical behaviour of composite media.

Key words: Composite materials, Foldy's theory, porous materials, Biot-Stoll model.

1. INTRODUCTION

Composite materials are being used increasingly as primary structural components in modern applications. The fracture and failure properties of these materials depend strongly on the state of local stress and deformations at or near the various interfaces, and the capability to predict these quantities accurately is essential for both conventional and damage tolerant design of the structure. However, it is virtually impossible to apply conventional techniques to solve stress analysis for realistic models incorporating their detailed microstructure. Thus attention has been focused on obtaining solutions based on simplified models.

In one of the most widely used approximate techniques, the microstructure is assumed to be smeared out, resulting in a homogeneous material with certain "effective" elastic properties. This technique has been widely used for materials containing a random distribution of canonical scatterers, such as spheres or cylinders.

The waves propagating in such a medium undergo multiple scattering which results in a frequency dependent velocity and attenuation of the coherent waves.

The problem of the propagation of multiple scattered waves in a random distribution of objects has been studied in the literature for many years. Foldy [1],

* Paper presented at the National Conference on Applied Physics, June 9–10, 2006, Galați, Romania

Watterman and Truell [2] and Twersky [3] have developed statistical procedures for estimating the dynamic properties of the average (coherent) waves in terms of the concentration of scatterers.

In these methods, the multiple scattering formula yields the propagation constant of the average waves in terms of the forward and backward far-field amplitude for an isolated scatterer, and the results are accurate provided the concentration of scatterers is not too high.

In order to apply these theories it is useful to study the acoustic scattering on an isolated scatterer. In this paper the scatterer is a fluid (air) saturated porous cylinder in an elastic matrix. This may represent a first step towards the understanding of the ultrasound propagation in an aging material with porous regions.

In Foldy's theory the backward scattering is neglected compared to the forward scattering term. Our aim is to see at which conditions such an approximation is valid in a composite matrix with porosity. The scatterer, in that case, is supposed to be an infinitely long air-filled porous cylinder, which solid matrix is the same one as the surrounding (composite) medium.

2. BIOT MODELIZATION OF A POROUS MEDIUM

Biot's [4] theory of fluid saturated porous media provides a continuum theory which permits the fluid and solid components to move independently and accounts approximately for the attenuation of waves due to the viscous friction.

Stoll's model [5] extends it to cases where viscoelasticity of the solid frame should also be taken into account in order to describe the actually observed attenuation in a given porous medium.

The elastic composite is characterized by c_p , c_s and ρ (velocities of compression and shear waves and density). The porous cylinder, of radius a , is supposed infinitely long in the z direction. It is characterized by porosity β , density ρ of the solid, density ρ_f of the saturating fluid, structure factor α , bulk moduli K_f of the fluid, K_s of the solid and K_b of the solid frame (dried porous medium), and the shear modulus μ of the frame. The solid frame in the case we study is epoxy resin, as well as the surrounding "composite", and the pores are air-filled.

Acoustic scattering by a cylindrical target is a procedure quite well known, so we will not detail it [6, 7]. The boundary conditions (closed pores) at the porous cylinder/elastic composite interface have been described by Deresiewicz and Skalak [8].

We defined the scattering S matrix in order to verify its unitarity as predicted by energy conservation. The S matrix is a diagonal block matrix, as conversions from mode n to mode $m \neq n$ do not occur. The diagonal block terms are 2×2 matrices:

$$S_n = \begin{pmatrix} S_n^{PP} & S_n^{PS} \\ S_n^{SP} & S_n^{SS} \end{pmatrix} \quad (1)$$

with S_n^{ij} the amplitude of the scattered wave of type i , $i = P$ (compression), S (shear), when the incident wave is a converging Hankel function $H_n^{(2)}$ of type j ($j = P, S$).

Each S_n matrix verify:

$$S_n^\dagger \cdot S_n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2)$$

with \dagger indicating transposition and complex conjugation.

3. FOLDY' THEORY

For a harmonic incident plane wave, the far field form function is classically defined as:

$$f = \frac{2}{\sqrt{\pi \cdot x}} \left| \sum_{n=-\infty}^{\infty} T_n \cdot e^{in\theta} \right| \quad (3)$$

where T_n is the amplitude matrix defined by

$$T = T_n = \frac{1}{2i} (S_n - I), \quad (4)$$

x the reduced frequency of the incident wave, and $\theta = 0$ for forward scattering and $\theta = \pi$ for backward scattering.

As conversions from mode n to mode $m \neq n$ do not occur only f^{PP} and f^{SS} are to be calculated.

The effective complex wavenumbers of the multiple scattered waves can be obtained in terms of frequency and the structure. For multiple scattering by a random distribution of cylindrical scatterers, Foldy's theory derived expressions for complex wavenumbers:

$$k_{Pef} = k_L + \frac{2n_0}{k_L} f^{PP}(0) \quad (5)$$

and

$$k_{Sef} = k_S + \frac{2n_0}{k_S} f^{SS}(0) \quad (6)$$

with n_0 – density of the scatterers and $f^{PP}(0)$, $f^{SS}(0)$ are the forward scattering amplitudes from an isolated scatterer contained in the matrix.

From these relations it can be seen that the backward scattering amplitudes can be neglected. This fact must be numerically verified.

4. RESULTS AND DISCUSSION

In Figs. 1 and 2 the forward and backward far-field form functions are plotted for porosity values of respectively 0.1 and 0.001. It can be shown that at high frequencies the backward scattering may be neglected. More precisely if the frequency is greater than a critical frequency, defined by Johnson *et al.* [9] as the transition between the diffusive and propagation regimes of the slow wave in the porous medium, the approximation used by Foldy is valid.

Relations (5) and (6) are to be used in order to describe the scattering by the “effective” medium.

In order to confirm the validity of our model a comparison between the dispersion curves obtained with our model and those obtained using the complicated Biot-Stoll model was made. In Figs. 3 and 4 numerical results are presented.

It can be seen a very good agreement between the numerical results. This fact permit the study of a complicated scattering process by simple means.

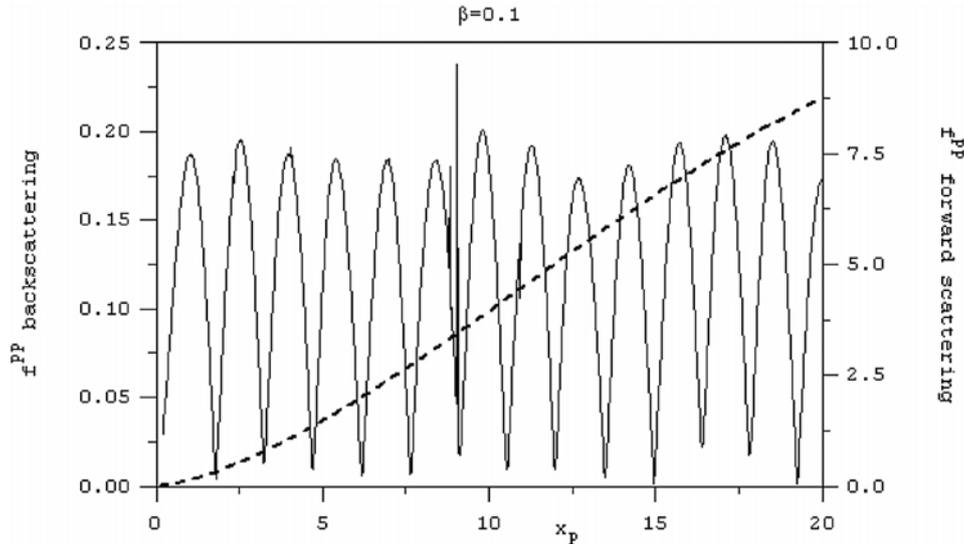


Fig. 1 – Forward (solid) and backward (dotted) scattering amplitudes for high porosity ($\beta = 0,1$).

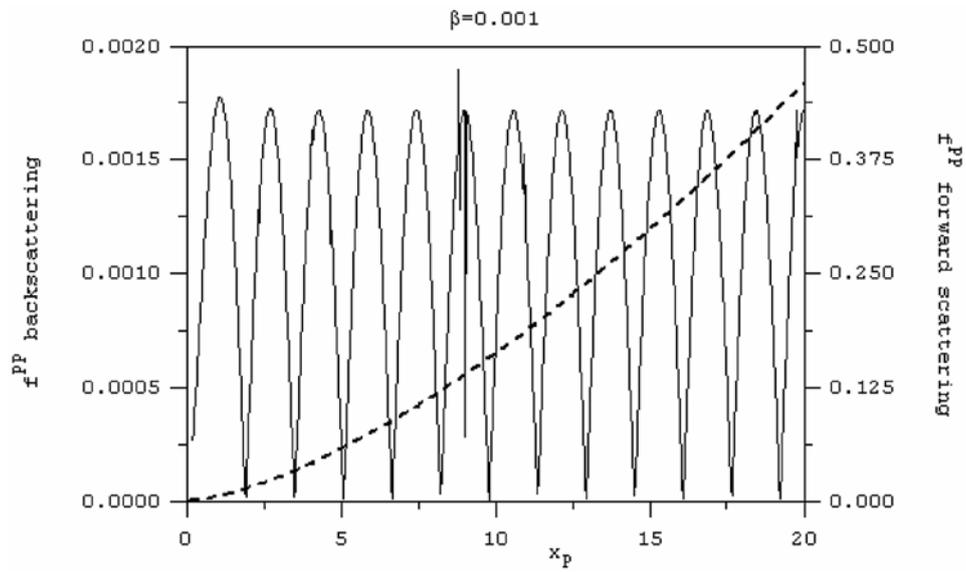


Fig. 2 – Forward (solid) and backward (dotted) scattering amplitudes for low porosity ($\beta = 0,001$).

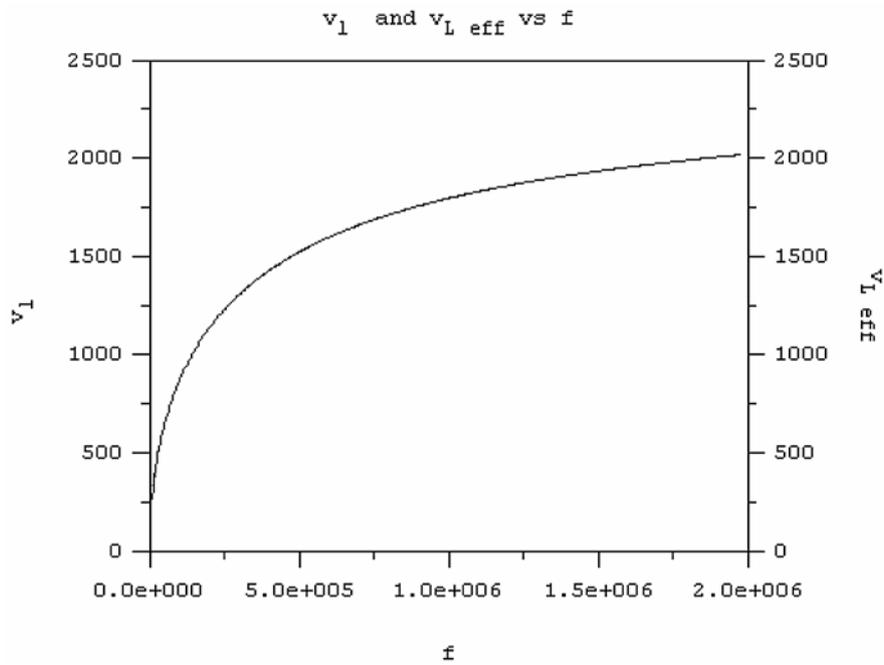


Fig. 3 – Comparison in dispersion curves between the two models.

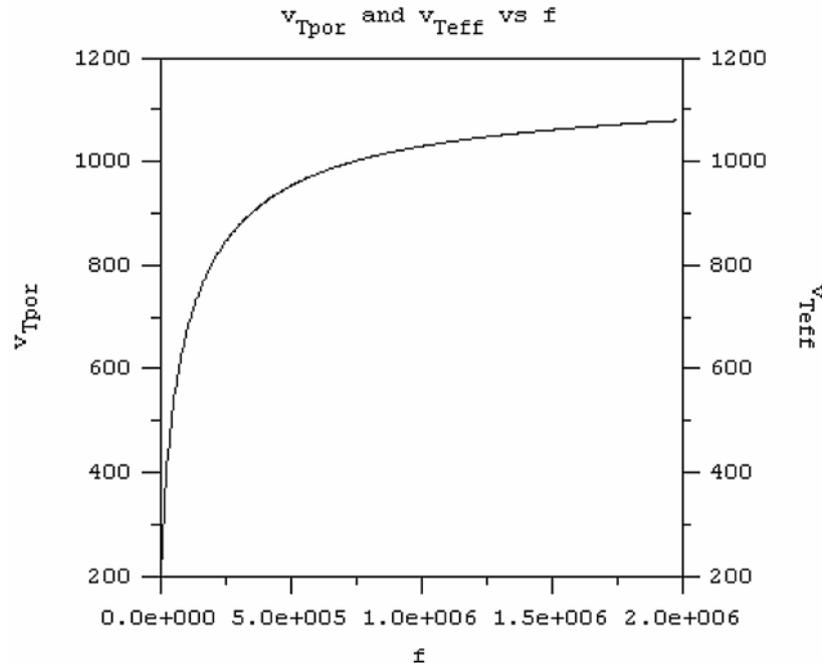


Fig. 4 – Comparison in dispersion curves between the two models.

5. CONCLUSIONS

The study of mechanical behaviour of composite media by means of acoustic scattering is used from long time. Usually in this kind of study, these composite media are replaced by effective ones. This work is a first step in the study of porous inclusions in a composite medium.

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