THE LINEAR NUMERICAL ANALYSIS OF DISPLACEMENT RESPONSE AMPLITUDE OPERATOR, BASED ON THE HYDROELASTICITY THEORY, FOR A BARGE TEST SHIP★

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In this paper there is presented the numerical analysis for the vertical displacement response amplitude operator of a barge test ship, based on the linear-modal analysis and the hydroelasticity theory. The solution of motion equations system is obtained in the frequency domain. The test ship and experimental data are supported by the Bureau Veritas Register in the frame of EU-FP6 Marstruct Project. The eigen numerical results are in good agreement with the experimental data.

Key words: ships hydroelasticity, linear numerical analysis, response amplitude operator.

1. INTRODUCTION

This study is focused on the linear-modal analysis of the ships dynamic response in head waves at zero speed, based on the hydroelasticity theory. The test ship is a barge with two constructive variants, prismatic (barge 1) and with small changes in the fore pick (barge 2), proposed by the Bureau Veritas Register [5], as comparative data. The numerical analysis is carried on with eigen programs pack DYN-LIN, module HEL [3].

2. THE THEORETICAL MODEL FOR LINEAR DYNAMIC SHIP RESPONSE, BASED ON HYDROELASTICITY

2.1 THE HYPOTHESES

The *springing phenomenon* it represents a steady state dynamic response produced at the resonance between the external wave excitation and the ship elastic girder eigen vibration modes [1].

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1) The ship hull girder is modelled using the finite element method (FEM), with Ne Timoshenko beam elements [2, 3], including bending and shearing deformations in vertical plane.

2) The hydrodynamic forces $F_h(x, t)$ are based on the strip theory, Gerritsma & Beukelman model [1, 3], that are functions of the elastic ship response, according to hydroelasticity theory.

3) Based on the modal analysis technique, it is considered the ship dynamic response decomposed on $r = 0, 1$ oscillations modes and $r = 2, n$ the first eigen vibration modes of the elastic ship girder.

4) It is considered an external excitation linear head wave, Airy model [1], and zero ship speed u_s.

5) The hydrodynamic masses $m_{33}(x)$ and damping coefficients $N_{33}(x)$ will be constant in time, calculated at the wave circular frequency ω, for still water position ($b(x)$ water-line breadth), Lewis transversal sections forms [7], based on Vugts method [6].

2.2. THE MOTION DIFFERENTIAL EQUATIONS IN VERTICAL PLANE

According the 3-rd hypotheses, the ship steady state linear dynamic response is decomposed in: $x \in [0, L]$

$$w(x,t) = \sum_{r=0}^{n} w_r(x) p_r(t); \quad \theta(x,t) = \sum_{r=0}^{n} \theta_r(x) p_r(t); \quad \gamma(x,t) = \sum_{r=0}^{n} \gamma_r(x) p_r(t) \quad (1)$$

where: $w_r(x)$, $\theta_r(x)$, $\gamma_r(x)$ are the modal form functions; $p_r(t)$ are the linear principal modal coordinates; L is the ships length.

Based on the 1-st and 3-rd hypotheses, the linear motion equations system is:

$$[a]\{\ddot{p}(t)\} + [b]\{\dot{p}(t)\} + [c]\{p(t)\} = \{F_h(t)\}$$

$$a_{ss} = \int_0^L \left[\mu(x) w_s^2(x) + j_y(x)\theta_s^2(x)\right] dx; \qquad r, s = 0, n$$

$$b_{rs} = \int_0^L \left[\alpha_r(x) GA_{fz}(x)\gamma_r(x)\gamma_s(x) + \beta_r(x) EI_y(x)\theta'_r(x)\theta'_s(x)\right] dx \quad (2)$$

$$c_{ss} = \int_0^L \left[EI_y(x)\left(\theta'_s(x)\right)^2 + GA_{fz}(x)\gamma_s^2(x)\right] dx; \quad F_{hs}(t) = \int_0^L F_h(x,t) w_s(x) dx$$

where: μ, j_y, EI_y, GA_{fz} the inertial and rigidity characteristics; α, β the structural damping coefficients.

The structural damping coefficients, according to Johnson & Tamita [1], are the following:

$$\alpha_r(x) \approx \beta_r(x) = 0.001 \cdot \Gamma_r; \quad \Gamma_0 = \Gamma_1 = 0; \quad \Gamma_2 = 1; \quad \Gamma_3 = 0.95; \quad \Gamma_4 = 0.9 \tag{3}$$

Base on the 4-th hypothesis, the linear equivalent wave, Airy model [1], has the elevation real and complex forms as following:

$$\overline{\zeta}_v^*(x,t) = a_w e^{-kT}\varepsilon(x)\cos(kx+\omega t) \quad \text{or} \quad \overline{\zeta}_v^*(x,t) = a_w e^{-kT}\varepsilon(x)e^{i(kx+\omega t)} \tag{4}$$

where: a_w the wave amplitude; e^{-kT} the Smith factor; $\varepsilon(x)$ the average coefficient over the ship breadth.

Base on 2-nd and 5-th hypotheses, the hydrodynamic force has the expression:

$$\begin{aligned} &F_h(x,t) = -\left\{\frac{\partial}{\partial t}\left[m_{33}(x)\frac{\partial z_r}{\partial t}\right] + N_{33}(x)\frac{\partial z_r}{\partial t} + \rho g b(x) z_r\right\}; \\ &z_r(x,t) = w(x,t) - \overline{\zeta}_v^*(x,t) \end{aligned} \tag{5}$$

From relations (1)–(5) the motion equations system has the next matrix form:

$$\begin{aligned} &[A(\omega)]\{\ddot{p}(t)\} + [B(\omega)]\{\dot{p}(t)\} + [C(\omega)]\{p(t)\} = \{F_w(t)\} = \\ &\quad = [\{F_1\} + i\{F_2\}]a_w e^{-i\omega t} \\ &[A(\omega)] = [a] + [A_h(\omega)]; \quad [B(\omega)] = [b] + [B_h(\omega)]; \\ &\quad [C(\omega)] = [c] + [C_h(\omega)] \\ &A_{hrs} = \int_0^L m_{33}(x) w_r(x) w_s(x)dx; \quad B_{hrs} = \int_0^L N_{33}(x) w_r(x) w_s(x)dx; \\ &C_{hrs} = \int_0^L \rho g b(x) w_r(x) w_s(x)dx; \quad F_{ws}(t) = \int_0^L F_w(x,t) w_s(x)dx; \\ &F_w(x,t) = m_{33}(x)\frac{\partial^2 \overline{\zeta}_v^*(x,t)}{\partial t^2} + N_{33}(x)\frac{\partial \overline{\zeta}_v^*(x,t)}{\partial t} + \rho g b(x)\overline{\zeta}_v^*(x,t) \end{aligned} \tag{6}$$

2.3. THE DYNAMIC RESPONSE AMPLITUDE OPERATORS RAO

From relation (6) it results an equivalent linear algebraic system, as following:

$$\begin{aligned} &\{p(t)\} = [\{p_1\} + i\{p_2\}]e^{-i\omega t} \Rightarrow [D(\omega)]\{\overline{p}\} = \{\overline{F}\}; \quad \{\overline{p}\} = \{\{p_1\},\{p_2\}\}^T; \\ &\{\overline{F}\} = \{\{F_1\},\{F_2\}\}^T; [D(\omega)] = \begin{bmatrix} [C(\omega)] - \omega^2[A(\omega)] & \omega[B(\omega)] \\ -\omega[B(\omega)] & [C(\omega)] - \omega^2[A(\omega)] \end{bmatrix} \end{aligned} \tag{7}$$

Because the modal serial includes only the first 5 modes $r = 0,4$ ($n = 4$), the linear algebraic system (7) has the dimension 10×10, that can be solved using a standard Gauss library procedure [3].

The solution of system (7), for $a_w = 1$, $\omega = 0 \div 10$ rad/s, it is used for the calculation of the ships displacement dynamic response amplitude operator RAO and for short time prediction statistics.

$$w(x,t) = \mathrm{Re}\left\{\sum_{r=0}^{4} w_r(x)\left(p_{1r} + ip_{2r}\right)e^{-i\omega t}\right\} = w^1(x,\omega)\cos\omega t + w^2(x,\omega)\sin\omega t$$
$$RAO_w(x,\omega) = \left[w^1(x,\omega)/a_w\right]^2 + \left[w^2(x,\omega)/a_w\right]^2 \quad (8)$$

3. THE NUMERICAL ANALYSIS

3.1. THE INPUT DATA FOR THE BARGE MODELS

There are considered two model cases (barge 1, barge 2) for the barge test ship, as there are defined in the Bureau Veritas report [5] (see Fig. 1.1–2). The segments of the barge model are interconnected by two elastic steel plates. The elastic plates have 50 mm width and the thickness: 4 mm in first case barge 1 and 6mm in the second case barge 2. The equivalent input data for the barge models used in the DYN-LIN / HEL analysis are presented in the Table 1 [4]. The models mass and the mass inertial moment are considered uniform distributed over the ship length.

In Table 2 there are presented the oscillations and vibrations eigen circular frequencies values, obtained from analytical approaches and the Finite Element Method (FEM), with Timoshenko beam elements [4]. In Fig. 2.1–2 the eigen modes at oscillations and vibrations are presented for the two barge test models.

Table 1

Input equivalent data for the barge test ship models

Model	**Barge 1 (C1) Fig. 1.1**	**Barge 2 (C2) Fig. 1.2**
Ne FEM beam elements	30	38
D.O.F.	62	78
u_s [m/s]	0	0
wave	head	head
L [m]	2.445	2.445
B [m]	0.600	0.600
D [m]	0.250	0.250

Table 1 (continued)

Model	**Barge 1 (C1) Fig. 1.1**	**Barge 2 (C2) Fig. 1.2**
d [m]	0.120	0.120
d_{aft} [m]	0.120	0.11316
d_{fore} [m]	0.120	0.12691
c_B	1.00	0.98
ρ [kg/m^3]	1000	1000
Δ [kg]	176.04	172.53
b_s [mm] steel plate	50	50
t_s [mm] steel plate	4	6
A [m^2]	4.000E-04	6.000E-04
A_{fz} [m^2]	3.333E-04	5.000E-04
I_y [m^4]	5.333E-10	1.800E-09
μ [kg/m]	72.000	70.564
j_y [kgm^2/m]	4.11E-06	1.39E-05
E [N/m^2]	2.06E+11	2.06E+11
ν	0.3	0.3
G [N/m^2]	7.92E+10	7.92E+10
g [m/s^2]	9.81	9.81
ρ_m [kg/m^3]	7.70E+03	7.70E+03
dx [m] (element length)	0.0815	0.0815 / 0.0545 / 0.019

Table 2

The eigen circular frequencies of the barge test ship models [rad/s]

Model		Barge 1 (C1) Fig. 2.1		Barge 2 (C2) Fig. 2.2	
Analysis		Analytical	FEM	Analytical	FEM
$\omega_{heave,\ pitch}$	wet	5.595	–	5.617	–
ω_{flex1}	dry	4.633	4.623	8.597	8.579
	wet	3.123	3.117	5.763	5.771
ω_{flex2}	dry	12.749	12.743	23.660	23.648
	wet	9.229	9.225	17.045	17.070
ω_{flex3}	dry	24.980	24.982	46.356	46.358
	wet	18.811	18.818	34.765	34.810

3.2. RESPONSE AMPLITUDE OPERATORS RAO FOR BARGE MODELS

Based on the theoretical model presented in chapter 2, there are obtained the RAO functions for the total vertical displacement plus deformations and the principal modal coordinates (heave, pitch, the first, second and third vibration modes). The springing phenomenon is obtained on flex1 mode. There is considered the linear modal analysis, in the case with zero ship speed, head wave model Airy [1]. The hydrodynamic forces are calculated using strip theory. The wave circular frequency domain is $\omega = 0\text{–}10$ [rad/s] and step $\delta\omega = 0.005$ [rad/s]. In the following we present the numerical results obtained with DYN-LIN / HEL analysis [4]:

• Fig. 3.1–2 the RAO functions for the principal modal coordinates at models barge 1 and barge 2;

• Fig. 4.1–2 the RAO functions for the total vertical displacement and deformation at models barge 1 and barge 2, at main section x [m] = 1.215 m.

From the Bureau Veritas report [5], there are obtained comparisons data between several numerical and experimental results developed at Task Wave Induced Loads, in the frame of EU-FP6 Marstruct Project.

In the following we present results only for case barge 1, where there are noted the Bureau Veritas results by C1_BV & Expe and with UGAL our numerical results:

• Fig. 5.1–3 the RAO functions for the principal modal coordinates at model barge 1: heave, pitch and first vibration mode [5];

• Fig. 6 the RAO function for the total vertical displacement and deformation at model barge 1, at the main section x [m] = 1.215 m [5].

4. CONCLUSIONS

The numerical RAO functions, obtained in this study, have very close values to those presented in Bureau Veritas report [5], for the total vertical displacement (with deformations) RAO_w and for the principal modal coordinates RAO_{pr}, r = heave, pitch, flex1, 2, 3 (see Fig. 5.1–3 & 6).

The differences that occur have the following main sources:

- the precision of the input data idealization used in the tests;
- the structural damping coefficients are based on empiric values;
- method induced differences, because this study it is based on the 2D flow approach (strip theory).

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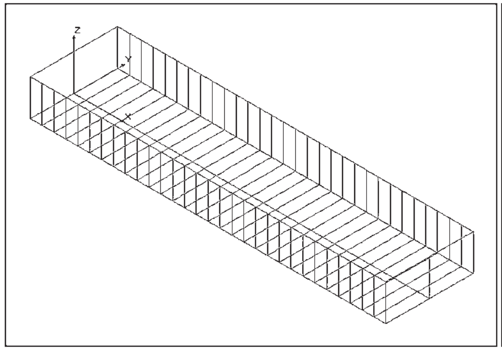

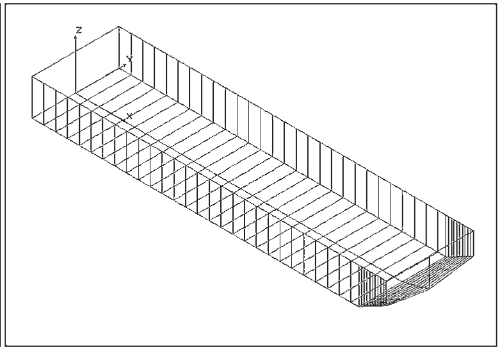

Fig. 1.1 – Barge 1 Model used in DYN-LIN / HEL analysis.

Fig. 1.2 – Barge 2 Model used in DYN-LIN / HEL analysis.

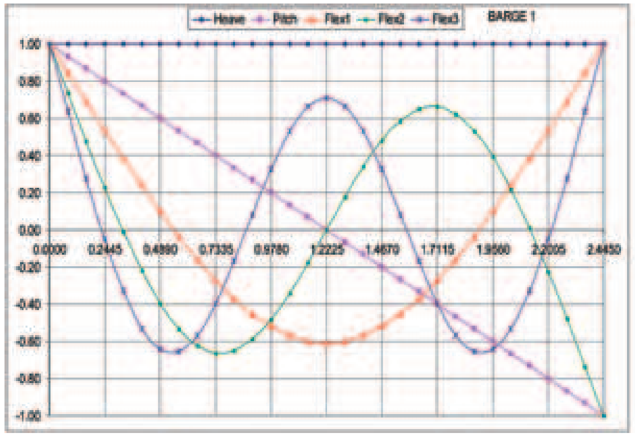

Fig. 2.1 – Eigen modes (dry), case barge 1 model.

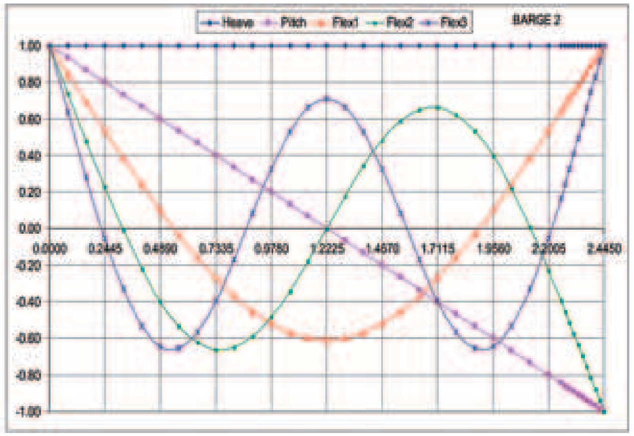

Fig. 2.2 – Eigen modes (dry), case barge 2 model.

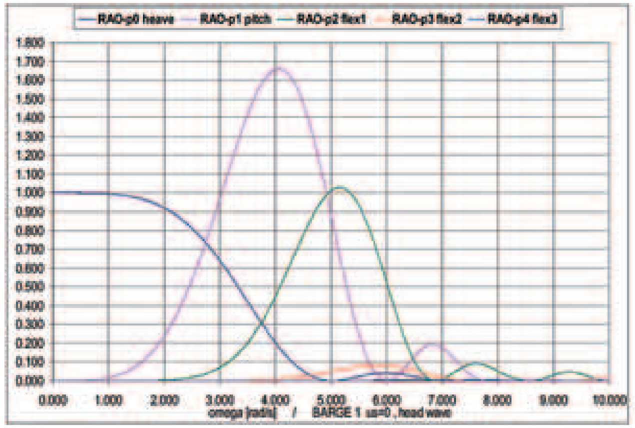

Fig. 3.1 – RAO for the principal modal coordinates, barge 1.

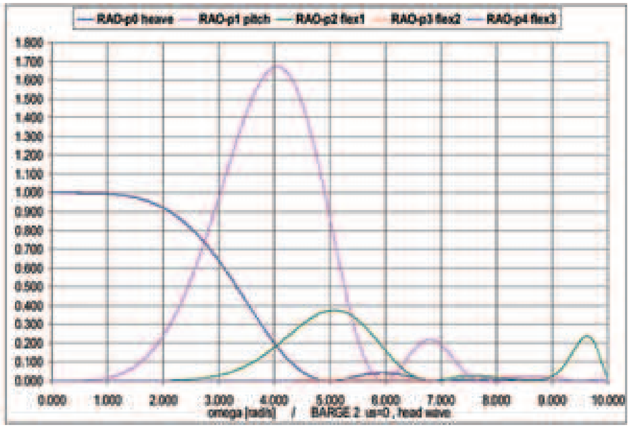

Fig. 3.2 – RAO for the principal modal coordinates, barge 2.

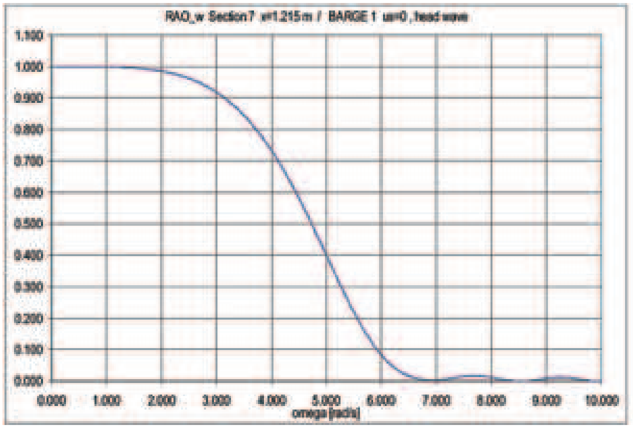

Fig. 4.1 – RAO total vertical displacement, barge 1 model, x [m] = 1.215 m.

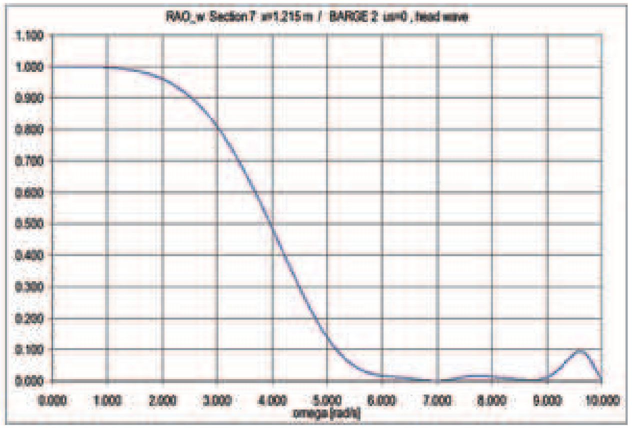

Fig. 4.2 – RAO total vertical displacement, barge 2 model , x [m] = 1.215 m.

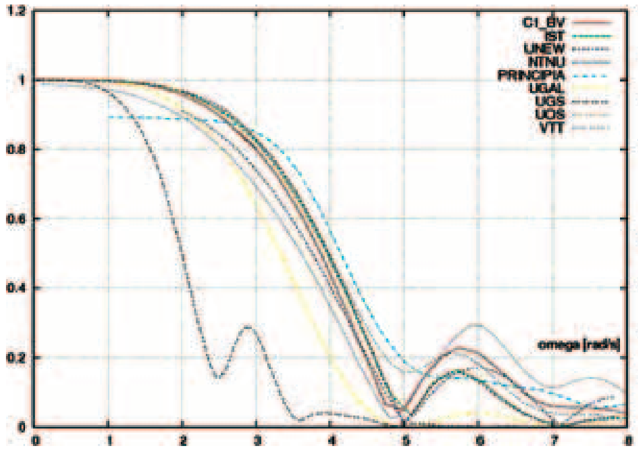

Fig. 5.1 – The RAO function for the principal modal coordinate at heave, barge 1 model [5].

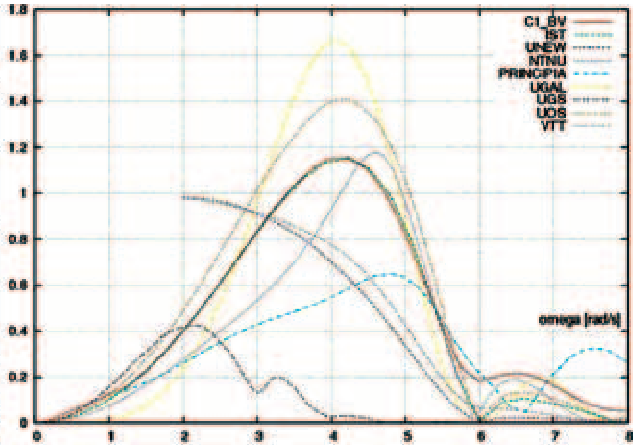

Fig. 5.2 – The RAO function for the principal modal coordinate at pitch, barge 1 model [5].

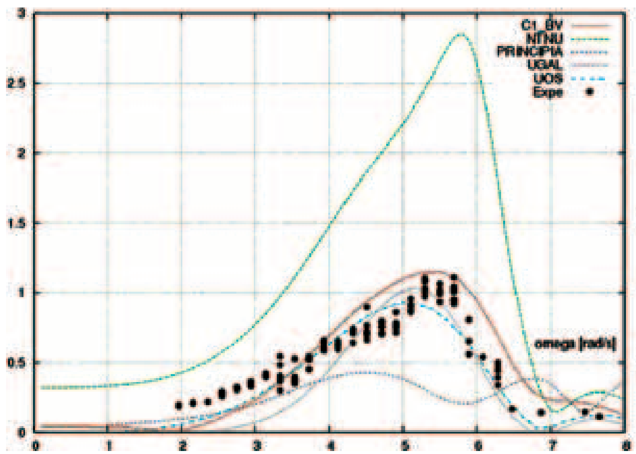

Fig. 5.3 – The RAO function for the principal modal coordinates at first vibration mode, barge 1 model [5].

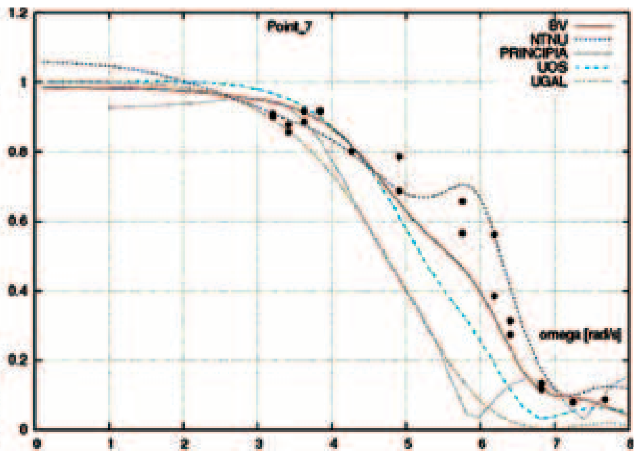

Fig. 6 – The RAO function for the total displacement and deformation, model barge 1, at section x [m] = 1.215 m [5].