

# APPLYING THE KKW GENERALIZED ANALYSIS TO A MAGNETIC STRINGY SOLUTION\*

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We apply the Keski-Vakkuri, Kraus and Wilczek (KKW) generalized analysis to a magnetic stringy black hole solution to compute its temperature and entropy. The solution that we choose in the Einstein-dilaton-Maxwell theory is the dual solution known as the magnetic black hole solution.

Our results show that the expressions of the temperature and entropy of this non-Schwarzschild-type black hole are not the Hawking temperature and the Bekenstein-Hawking entropy, respectively. In addition, the extremal magnetic stringy black hole is not frozen because it has a constant non-zero temperature.

*Key words:* KKW analysis, magnetic stringy black hole.

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## INTRODUCTION

As we know from the literature, the Keski-Vakkuri, Kraus and Wilczek (KKW) analysis [1] has been used to studying the Schwarzschild-type black hole solution [2] and, after this, to compute the temperature and entropy of other black hole space-times [3–5]. In this analysis, the total Arnowitt-Desser-Misner mass [6] is fixed but the mass of the Schwarzschild black hole decreases due to the emitted radiation. To avoid the singularities at the horizon the Painlevé [7] coordinate transformation is used and this enable us the study of the across-horizon physics such as the black hole radiation. The black hole temperature depends not only on the characteristics of the black hole but, also, on the energy of the emitted shell of energy. Furthermore, the black hole entropy is not given by Bekenstein and Hawking formula for the specific black hole.

For studying the temperature and entropy of a black hole solution which is not of Schwarzschild-type we can apply successfully the (KKW) generalized analysis. This generalized (KKW) analysis case was studied by Vagenas [8] and he established the formulas for the temperature and entropy of a black hole

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solution described by a metric which satisfies  $A(r) \cdot B^{-1}(r) \neq 1$ . The Hawking radiation is viewed as a tunneling process which emanates from the non-Schwarzschild-type black hole solution. Vagenas [8] introduced a more general coordinate transformation in order to apply the (KKW) analysis to non-Schwarzschild-type black holes. He established two conditions: 1) the regularity at the horizon which ensures that we can study the across-horizon physics and 2) the stationarity of the non-static metric which implies that the time direction is a Killing vector, condition that is very important to generalize the (KKW) analysis. The results of this generalized analysis are the exact expressions of the temperature and entropy of the non-Schwarzschild-type black holes which are not the Hawking temperature  $T_H$  and the Bekenstein-Hawking entropy  $S_{BH}$ .

In this paper we apply the Keski-Vakkuri, Kraus and Wilczek (KKW) generalized analysis to a magnetic stringy black hole solution [9] in order to evaluate the temperature and entropy of this black hole space-time. The solution that we study is one of the Einstein-dilaton-Maxwell theory, the dual solution known as the magnetic black hole solution. The metric is obtained by multiplying the electric metric in the Einstein frame by a factor  $e^{2\Phi}$ .

#### KKW GENERALIZED ANALYSIS FOR THE MAGNETIC STRINGY BLACK HOLE

String theory may be the best way to attain the holy grail of fundamental physics, which is to generate all matter and forces of nature from one basic building block. Through the years, many important studies have been made related to the string theory.

The low-energy effective theory largely resembles general relativity with some new “matter” fields as the dilaton, axion etc [9–10]. A main property of the low-energy theory is that there are two different frames in which the features of the space-time may look very different. These two frames are the Einstein frame and the string frame and they are related to each other by a conformal transformation ( $g_{\mu\nu}^E = e^{-2\Phi} g_{\mu\nu}^S$ ) which involves the massless dilaton field as the conformal factor. Even the existence of these two different frames makes the difference between the two theories, the low energy effective theory and the Einstein theory. The string frame is actually similar to the Brans-Dicke frame in the Jordan-Brans-Dicke theory. The string “sees” the string metric. Many of the important symmetries of string theory also rely of the string frame or the Einstein frame. The  $T$  duality [11] transformation relates metrics in the string frame only, whereas  $S$  duality [12] is a valid symmetry only if the equations are written in the Einstein frame. Kar [9] gave important results about the stringy black holes and energy conditions. There were studied several black holes in two

and four dimensions with regard to the Weak Energy Conditions (WEC). It is very important to study the black holes in string theory and this can be explained in the following way. Because string theory is expected to provide us with a finite and clearly defined theory of quantum gravity, the answer of many questions related with black hole evaporation could be solved in the context of string theory. To do this, is required to construct black hole solutions in string theory.

The action for the Einstein-dilaton-Maxwell theory in 3+1 dimensions is given by

$$S_{EDM} = \int d^4x \sqrt{-g} e^{-2\Phi} \left[ R + 4g_{ik} \nabla^i \Phi \nabla^k \Phi - \frac{1}{2} g^{ij} g^{kl} F_{ik} F_{jl} \right]. \quad (1)$$

Varying with respect to the metric, dilaton and Maxwell fields we get the field equations for the theory given as

$$R_{ik} = -2\nabla_i \nabla_k \Phi + 2F_{ij} F_k^j, \quad (2)$$

$$\nabla^k (e^{-2\Phi} F_{ik}) = 0, \quad (3)$$

$$4\nabla^2 \Phi - 4(\nabla \Phi)^2 + R - F^2 = 0. \quad (4)$$

The metric (in the string frame) which solve the Einstein-dilaton-Maxwell field equations to yield the electric black hole is given by

$$ds^2 = -A \left( 1 + \frac{2M \sinh^2 \alpha}{r} \right)^{-2} dt^2 + \frac{1}{A} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \quad (5)$$

where  $A = 1 - 2M/r$ .

In the string frame the dual solution known as the magnetic black hole is obtained by multiplying the electric metric in the Einstein frame by a factor  $e^{-2\Phi}$ . This is in a generalized sense even the  $S$ -duality transformation which changes  $\Phi \rightarrow -\Phi$  and thereby inverts the strength of the string coupling. We mention that the magnetic and electric solutions are the same if looked at from the Einstein frame. Therefore, the magnetic black hole metric is given by

$$ds^2 = -\frac{A}{B} dt^2 + \frac{1}{AB} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \quad (6)$$

where

$$B = 1 - \frac{Q^2}{Mr}. \quad (7)$$

The metric describes a black hole with an event horizon at  $r_+ = 2M$ .

The (KKW) methodology generalized by Vagenas [8] in the case of the non-Schwarzschild-type black hole geometries requires that  $A(r) \cdot B^{-1}(r) \neq 1$ ,

condition that is satisfied by the solution given by (6) and (7). Also, the total Arnowitt-Desser-Misner mass  $M_{ADM}$  have to be well-defined so we get  $A(r) \rightarrow 1$ , as  $r \rightarrow \infty$  and  $B(r) \rightarrow 1$ , as  $r \rightarrow \infty$ . We need to have the regularity at the event horizon and, also, the stationarity of the non-static metric which implies that the time direction is a Killing vector [13–14]. The Painlevé [7] more general coordinate transformation is given by

$$\sqrt{A(r)}dt = \sqrt{A(r)}d\tau - \sqrt{B^{-1}(r)-1}dr, \quad (8)$$

where  $\tau$  is the new time coordinate. The metric given by (6) becomes

$$ds^2 = -A(r)d\tau^2 + 2\sqrt{\frac{A(r)}{B(r)}}(1-B(r))d\tau dr + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2. \quad (9)$$

The radial null geodesics are

$$\dot{r} = \sqrt{\frac{A(r)}{B(r)}} \left[ \pm 1 - \sqrt{1-B(r)} \right]. \quad (10)$$

In the equation above, the upper (lower) sign corresponds to the outgoing (ingoing) geodesics under the assumption that  $\tau$  increases towards future.

The total Arnowitt-Desser-Misner mass  $M_{ADM}$  is fixed and the mass  $M$  of the black hole fluctuates because a shell of energy  $\omega$  which constitutes of massless particle considering only the s-wave part of emission, is radiated by the black hole. Now, the massless particles travel on the outgoing geodesics which are due to the varying mass  $M$  of the black hole. The metric is given by

$$ds^2 = -A(r, M - \omega)d\tau^2 + 2\sqrt{\frac{A(r, M - \omega)}{B(r, M - \omega)}}(1-B(r, M - \omega))d\tau dr + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2. \quad (11)$$

We get that the outgoing radial null geodesics will have a new formula

$$\dot{r} = \sqrt{\frac{A(r, M - \omega)}{B(r, M - \omega)}} \left[ 1 - \sqrt{1-B(r, M - \omega)} \right], \quad (12)$$

using the modified outgoing geodesics and with

$$A' = A(r, M - \omega'), \quad B' = B(r, M - \omega') \quad (13)$$

the imaginary part of the action (see equation (32) in [8]) takes the form

$$\text{Im } I = \text{Im} \int_{r_+(M-\omega)}^{r_+(M)} \int_0^{+\omega} \frac{d\omega'}{\sqrt{\frac{A'}{B'}(1-\sqrt{1-B'})}} dr.. \quad (14)$$

Now, we calculate the temperature of the black hole (see equation (35) in [8]) and we get

$$T_{bh}(M, \omega) = \frac{\omega}{4\pi M^2 \left[ 1 - \left( 1 - \frac{\omega}{M} \right)^2 \right]}. \quad (15)$$

The expression of the entropy is given by

$$S_{bh} = S_{BH} - 4\pi M^2 \left[ 1 - \left( 1 - \frac{\omega}{M} \right)^2 \right]. \quad (16)$$

If we evaluate the  $T_{bh}$  temperature to first order in  $\omega$  we get the Hawking temperature  $T_H$  of the magnetic stringy black hole which is

$$T_H = \frac{1}{8\pi M}. \quad (17)$$

The entropy of the black hole is not the Bekenstein-Hawking entropy  $S_{BH}$

$$S_{BH} = \pi r_+^2 = 4\pi M^2. \quad (18)$$

Furthermore, the black hole entropy  $S_{bh}$  to zeroth order in  $\omega$  yields the Bekenstein-Hawking entropy  $S_{BH}$ . We make the calculations in a new framework, where the extremal magnetic stringy black hole which is created when we have

$$Q^2 = 2(M - \omega)^2, \quad (19)$$

so, the extremality condition  $r_+ = r_-$  is not the same and the temperature of the extremal black hole does not vanish and becomes

$$T_{bh}(M, Q) = \frac{1}{4\pi M \left( 1 + \frac{Q}{\sqrt{2}M} \right)}. \quad (20)$$

Also, the following condition has to be respect

$$Q < \sqrt{2}M, \quad (21)$$

because the emitted shell of energy  $\omega$  has to take only positive values. A naked singularity will not appear from the collapse of the magnetic stringy black hole.

## DISCUSSION

We used the (KKW) generalized analysis given introduced in [8] in the case of a non-Schwarzschild-type black hole solution which is a magnetic

stringy black hole solution in the Einstein-dilaton-Maxwell theory, in order to evaluate the temperature and entropy of this black hole space-time. The temperature and entropy of the black hole are different from the Hawking temperature  $T_H$  and the Bekenstein-Hawking entropy  $S_{BH}$ . The black hole temperature depends on the emitted massless particle's energy. The temperature and entropy of the black hole in the first and, respectively, zeroth orders of  $\omega$  correspond to the Hawking temperature  $T_H$  and the Bekenstein-Hawking entropy  $S_{BH}$ . These results sustain the validity of the generalized KKW analysis introduced by Vagenas [8].

We established that the extremal magnetic stringy black hole is not frozen. Also, it has a non-zero background temperature since the extremality condition was changed.

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