MODELING THE ENERGY DISTRIBUTIONS OF A DILATON-MAXWELL GRAVITY SOLUTION*

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In this paper we extend the investigations related to the energy localization and we move on towards modeling the connection between the energy distributions of a metric which describes a recently derived non-asymptotically flat black hole solution in dilaton-Maxwell gravity. Energy distributions are evaluated with the Møller and Landau and Lifshitz energy-momentum complexes. The energy distributions depend on the mass M, charge Q of the black hole, γ parameter and r coordinate. We established a connection between the terms of the ratio of the energy distributions in the Landau and Lifshitz and Møller prescriptions. All the terms contain the quantity $Q^n r^{n-1} M^{-n}$ and, also, a function $f(\gamma)$ which depends on the γ parameter. The Landau and Lifshitz and Møller prescriptions turn out to be powerful tools for energy-localization for various physical systems.

Key words: modeling, energy distribution, dilaton-Maxwell gravity solution.

INTRODUCTION

One of the most interesting problems in General Relativity, energy-momentum localization is connected to the use of various energy-momentum complexes. Also, the subject of the localization of energy continues to be an open one since Einstein [1] has given his important result of the special theory of relativity that mass is equivalent to energy. The method of localization of energy by using several energy-momentum complexes has many adepts but there was, also, many criticism related to the use of these prescriptions. Their main lack is that most of these restrict one to calculate in quasi-Cartesian coordinates. Only the Møller energy-momentum complex allows us to make the calculations in any coordinate system. After the Einstein work [1–2], a plethora of energy-momentum complexes were constructed, including those of Einstein [1–2], Landau and Lifshitz [3], Papapetrou [4], Bergmann [5], Weinberg [6] (ELLPW) and Møller [7].

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This important issue, the energy-momentum localization by using the energy-momentum complexes was re-opened in the last two decades by Virbhadra and his collaborators and many interesting results have been obtained in this area [8]. Misner *et al.* [9] sustained that to look for a local energy-momentum means that is looking for the right answer to the wrong question. Also, they concluded that the energy is localizable only for spherical systems. Cooperstock and Sarracino [10] demonstrated that if the energy is localizable in spherical systems then it is also localizable in any space-times. Chang, Nester and Chen [11] showed that the energy-momentum complexes are actually quasilocal and legitimate expression for the energy-momentum. They concluded that there exist a direct relationship between energy-momentum complexes and quasilocal expressions because every energy-momentum complexes is associated with a legitimate Hamiltonian boundary term. Very important is the Cooperstock hypothesis [12] which states that energy and momentum are confined to the regions of non-vanishing energy-momentum tensor for the matter and all non-gravitational fields.

Regarding Møller's prescription, there are many good results [8] that recommend it as a reliable tool for energy-momentum localization. According to the Lessner opinion [13], Møller's energy-momentum complex is significant for describing the concepts of energy and momentum in General Relativity. He sustained that *The energy-momentum four-vector can transform according to special relativity only if it is transformed to a reference system with an everywhere constant velocity. This cannot be achieved by a global Lorentz transformation*.

As we pointed out, the energy-momentum complexes are powerful tools for energy-momentum localization. In this connection, we extend the investigations related to the energy localization and we move on towards modeling the connection between the energy distributions of a metric which describes a recently derived non-asymptotically flat black hole solution in dilaton-Maxwell gravity. Energy distributions are evaluated with the Møller and Landau and Lifshitz energy-momentum complexes. We established a connection between the terms of the ratio of the energy distributions in the Landau and Lifshitz and Møller prescriptions. Also, we make a comparison and we find some connections and differences between the coefficients in the Landau and Lifshitz and Møller prescriptions. Through the paper we use geometrized units ($G = 1$, $c = 1$) and follow the convention that Latin indices run from 0 to 3.

2. MODELING THE CONNECTIONS BETWEEN LANDAU AND LIFSHITZ AND MØLLER PRESCRIPTIONS

The low energy effective theory largely resembles general relativity with some new "matter" fields as the dilaton, axion etc. A main property of the low-energy theory is that there are two different frames in which the features of the space-time may look very different. These two frames are the Einstein frame and the string frame and they are related to each other by a conformal transformation $(g_{\mu\nu}^{E} = e^{-2\Phi} g_{\mu\nu}^{S})$ which involves the massless dilaton field as the conformal factor. The string “sees” the string metric. Many of the important symmetries of string theory also rely of the string frame or the Einstein frame.

Recently, a non-asymptotically flat black hole solution in dilaton-Maxwell gravity is due to Chan, Mann and Horne [14]. The string metric is given by

$$ds^2 = \frac{r^2 A}{\gamma^4} dt^2 - \frac{1}{A} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2, \tag{1}$$

where

$$A = 1 - \frac{2\sqrt{2}\,\gamma^2 M}{Qr} \tag{2}$$

and M and Q are the mass and the charge of the black hole. The energy distributions in the Landau and Lifshitz and Møller prescriptions are given by

$$E_{LL} = \frac{r}{2} \frac{2\sqrt{2}\gamma^2 M}{Qr - 2\sqrt{2}\,\gamma^2 M} \tag{3}$$

and

$$E_M = \frac{r^2}{\gamma^2} - \frac{\sqrt{2} M r}{Q}. \tag{4}$$

For $r \to \infty$ in the Landau and Lifshitz prescription we obtain $E_{LL} = \frac{\sqrt{2}\,\gamma^2 M}{Q}$ and in the case of the Møller prescription we have $E_M \to \infty$.

We established a connection between the terms of the ratio of the energy distributions in the Landau and Lifshitz and Møller prescriptions. This ratio is given by

$$E_{LL} / E_M = \frac{\sqrt{2} Q}{4M} + \frac{3Q^2 r}{8\gamma^2 M^2} + \frac{7\sqrt{2}\, Q^3 r^2}{32\gamma^4 M^3} + \frac{15 Q^4 r^3}{64\gamma^6 M^4} + \frac{31\sqrt{2}\, Q^5 r^4}{256\gamma^8 M^5}. \tag{5}$$

From equation (5) we conclude that at the denominators of these coefficients the power of γ coefficient increases with two units. At the numerators of this relation the r coordinate, charge Q and mass M respect the law of connection $Q^n r^{n-1} M^{-n}$ that is available for all the terms. For $r = 0$ we obtain

$$E_{LL} / E_M = \frac{\sqrt{2} Q}{4M}. \tag{6}$$

The ratio of the energy distributions can be written

$$E_{LL} / E_M = \sum_n a_n Q^n r^{n-1} M^{-n} f_n(\gamma). \tag{7}$$

Plotting (6) with the ratio of the energy distributions on z-axis against Q and M, we obtain the graph from the Fig. 1.

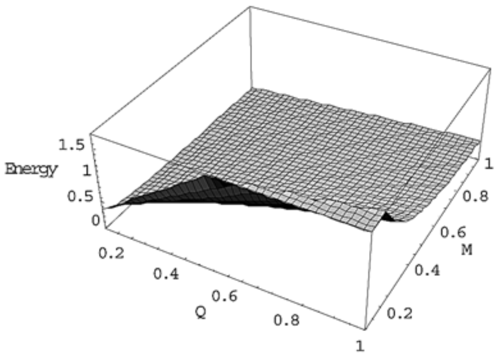

Fig. 1

The ratio of the energy distributions depends on the mass M, charge Q of the black hole, γ parameter and r coordinate. In the case $r = 0$ the ratio of the energy distributions depends on the mass M and charge Q of the black hole.

DISCUSSION

The important issue of energy localization still lacks of an acceptable answer and continues to be one of the most interesting and challenging problem of the General Relativity. We point out that this problem of defining in an acceptable manner the energy-momentum density hasn't got a generally accepted answer yet. The energy-momentum localization using several energy-momentum complexes (ELLPW) and Møller has many adepts but there was, also, much criticism related to the use of these prescriptions. The main lack of these prescriptions is that most of these restrict one to calculate in quasi-Cartesian coordinates. Only the Møller prescription enables one to make the calculations in any coordinate system.

Many researchers considered different space-times and compute the energy distribution using the energy-momentum complexes of Einstein, Landau and Lifshitz, Papapetrou, Bergmann, Weinberg (ELLPW) and Møller and obtained acceptable results. In many situations these prescriptions or some of them furnish the same result for the energy distribution of a given space-time. In some cases there were also obtained the same results for a given space-time using the definition of these energy-momentum complexes in both General Relativity and tele-parallel gravity. Chang, Nester and Chen [11] showed that the energy-momentum complexes are actually quasi-local and legitimate expressions for the energy-momentum. Their idea supports the energy-momentum complexes and the role which these are playing in energy-momentum localization. Furthermore, important studies have been done about the new idea of quasi-local approach for energy-momentum complexes [11, 15] and a large class of new pseudotensors connected to the positivity in small regions have been constructed [15]. In this light, the quasi-local quantities are associated with a closed 2-surface (L. B. Szabados, [15] and http://relativity.livingreviews.org/ Articles/lrr-2004-4/). The Hamiltonian boundary term determines the quasi-local quantities for finite regions and the special quasi-local energy-momentum boundary term expressions correspond each of them to a physically distinct and geometrically clear boundary condition [16].

We model the connection between the energy distributions of a metric which describes a recently derived non-asymptotically flat black hole solution in dilaton-Maxwell gravity. Energy distributions are calculated with the Møller and Landau and Lifshitz prescriptions.

We established a connection between the terms of the ratio of the energy distributions in the Landau and Lifshitz and Møller prescriptions. The ratio of the energy distributions depends on the charge Q, r coordinate and mass M of the black hole and contain the quantity $Q^n r^{n-1} M^{-n}$ and, also, a function $f_n(\gamma)$ which depends on the γ parameter. The dependence on the γ parameter is described by the quantity $\frac{1}{\gamma^{2m}}$, where $m = 0, 1, 2, \ldots$.

In a future work we intend to establish the connection between the energy distributions of this metric in the case of Einstein, Landau and Lifshitz and Møller prescriptions.

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