Dedicated to Acad. Aureliu Săndulescu's 75th Anniversary

NUCLEAR ASTROPHYSICS WITH RADIOACTIVE NUCLEAR BEAMS: INDIRECT METHODS

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The review describes a number of indirect methods in nuclear astrophysics using radioactive beams: Coulomb dissociation, transfer reactions (the ANC method), breakup of loosely bound nuclei at intermediate energies, the use other spectroscopic measurements, including β-decay studies and the determination of resonance parameters. The examples chosen are drawn from the experiments the author was involved in the last decade together with his group at Texas A&M University. One example discussed in particular is that of the reactions used to determine the S_{17} astrophysical factor for the $^{7}Be(p, \gamma)^{8}B$ reaction, crucial for the understanding the solar neutrino problem. We discuss also the case of the proton drip line nucleus ^{23}Al. From its study we extract data to determine the stellar reaction rates for $^{22}Mg(p, \gamma)^{23}Al$ and $^{22}Na(p, \gamma)^{23}Mg$. Both breakup and beta-decay methods were used in these latter studies.

Key words: nuclear physics for astrophysics, indirect methods, radioactive nuclear beams.

I. FOREWORD

This article is written following a lecture given at the Predeal International Summer School of Physics 2006, at the suggestion of the organizers. Essentially it follows the plan of that presentation, and is an extended version of the paper submitted for the proceedings of that school, but contains several additions, digressions, and more illustrations. I remembered that George Calinescu has once written [1] (I translate freely from memory): "once I have found a wording that I consider most appropriate, I did not find a reason to change it", and therefore, following this illustrious example, I have also included some passages from that work [2].

Given the purpose of this review, I have conceived it as addressed to a larger audience, mostly physics students or beginners in the field, making it more general and simplified rather than a report at a conference presenting latest personal results to a field of specialists, insisting on broader ideas than on technical details. The specificity will stem from that that, as illustrative examples, I shall use only experiments or studies to which I participated directly. Also, let me say that I would not ever claim that this is an exhaustive review of this rapidly evolving subject.

II. INTRODUCTION TO NUCLEAR PHYSICS FOR ASTROPHYSICS

Scientific knowledge about our Universe has evolved and evolves not only rapidly, but also very much (I mean quantity and quality) in the last hundred years or so, taking us to places which were not even dreamed by our predecessors not very long ago. And there is probably no domain in which this expansion of knowledge is more impressive than in understanding of the size, nature and composition of the physical Universe around us. The stars were for millennia objects of fascination for humans, but also an object of study, probably the very first objects of systematic studies. In time, watching them taught us many things, from measuring time, to orientation during travel, or to understanding our own smallness/greatness in the world in philosophical and poetical sense. However, most of the scientific information and the understanding that has come with it, have been acquired recently. Only at the middle of the XIX-th century the philosopher Auguste Comte was writing “we can never know anything of [stars’] chemical composition”. And how much do we know about the interior of the stars today, chemical composition in particular! All these because of a discipline that we call astrophysics. It extends simple astronomic observations with the methods of atomic physics, nuclear physics, radio-astronomy, gravitation ... We would go not only further in space with our observations (the size of the “known Universe” increased from around 100,000 light-years to over 13 billion light-years in the last 80 years alone, about 5 orders of magnitude in exactly the life span of my father!), but also deeper with our understanding. We see more with our instruments, which have taken us far beyond the margins of our own galaxy, and we “see” more with our understanding of our astrophysical measurements. The second half of the XX-th century took us as far as 13.7 billion light-years away, and the first years of this century took us even further. Further along the Copernican principle: he understood five hundred years ago that we are not in the center of the solar system, we knew for some time that we are not in the center of the Universe and we learn now we are not even made of the most abundant type of matter in the Universe [3]. Baryonic matter accounts for only a meager 4% of the total mass of the Universe, with the rest made of dark matter and dark energy, of yet obscure composition and origin!

Let us stick for now with these meager 4%! All chemical elements in the Universe as we know it (or rather, as we see it!) were produced in processes that we call generically nucleosynthesis. Nucleosynthesis occurred in various stages of the evolution of the Universe, in various places and in different types of events: Big Bang nucleosynthesis or later stellar evolution, far away or around us, explosive or steady burning. Space-based gamma-ray telescopes have had for some time the ability to detect γ-rays of cosmic origin. They have already provided strong and direct evidence that nucleosynthesis is an ongoing process in our Galaxy, close to where we live. Gamma-rays from the decay of long lived isotopes like ^{26}Al ($T_{1/2} = 7.2\cdot10^5$ y), ^{44}Ti (47.3 y), ^{56}Ni (6.1 d), etc. were detected. We also know today that the nuclear processes occurring in stars are not only the source of energy for cosmic processes, but also that nucleosynthesis gives us unique and indelible fingerprints of these processes. This is a very important point to understand the efforts in nuclear astrophysics today! There are several nucleosynthesis scenarios today, some which were formulated for some time (Big Bang Nucleosynthesis, Inhomogeneous Big Bang Nucleosynthesis, the s-process, the r-process, the rp-process, etc.), some which are newer proposals. The possibilities to check the predictions of specific models occurred only recently, with the availability of more nuclear data, of advances in understanding the dynamics of non-equilibrium processes, and of increased computing power. It turns out that an important component of all these nucleosynthesis model calculations is represented by the data for the nuclear processes involved. Only good nuclear physics data permit to make definite, quantitative predictions that can be checked against the ever increasing observational data sought and obtained by astrophysicists. The accumulation of these data (mostly cross sections and reaction rates, but also masses, lifetimes, nuclear level densities, GT strength distributions) is the object of the nuclear physics for astrophysics, a subject that we most often call nuclear astrophysics. The present review will not deal at all with specificities of the dynamics of different stellar processes, but only with the nuclear reactions involved, in particular with how we obtain these data from indirect measurements with radioactive nuclear beams (RNB). I shall also narrow the type of data sought to reaction cross sections and reaction rates.

There are many nuclear reactions and nuclear processes that occurred or occur in stars. For example, one important class is that of radiative capture reactions X(p, γ), X(n, γ), X(α, γ), etc. There are many problems we encounter in a nuclear astrophysics laboratory when we do direct measurements, by which we mean exactly the reactions that occur in stars, at stellar energies. Two main categories occur when we want to study the relevant reactions. One type of problems stems from the fact that the energies involved in stars are small (tens or hundreds of keV/u) and consequently the cross sections are extremely small in reactions involving charged particles, due to the Coulomb repulsion. This presents experimental challenges, related to beam intensities and background. As these reactions (radiative proton capture, radiative alpha capture, (α, p), (α, n) …) have cross sections of the order of nanobarns, picobarns, and even smaller at stellar energies, we can only make studies at somewhat larger energies (MeV/u, some few hundreds keV/u) and than extrapolate our measurements down to the relevant energy window (called Gamow peak). The other type or category of problems is that most of the reactions in stars involve short lived, unstable nuclei, produced in the preceding step of the stellar reaction chain. Therefore, many of the reactions cannot be measured with the stable beams and targets available in laboratory. These two major types of problems lead nuclear physicists to seek indirect methods for nuclear astrophysics and to the use of radioactive beams. Our subject today.

A number of specific notions are introduced in nuclear astrophysics: the astrophysical *S*-factor (to separate the Coulomb barrier penetration), the reaction rate (integrated over the Maxwellian distribution), the Gamow peak (the region in the Maxwellian energy distribution that contributes to the reaction rate). Let me give a few representative examples. In gases of temperature *T*, the average translational kinetic energy per particle is of the order of *kT*, where *k* is Boltzmann's constant. Expressing the temperature in million or billion kelvins ($T \equiv T_6 \cdot 10^6$ K, or $T \equiv T_9 \cdot 10^9$ K), as is usual in astrophysics, the average kinetic energy in keV is

$$\epsilon = \frac{3}{2}kT = \frac{3}{2}0.086T_6 \text{ keV} = \frac{3}{2}86T_9 \text{ keV} \tag{1}$$

This leads, for example for a temperature of the order of that in the center of the Sun $T_6 = 15$, to average energies of about 2 keV, or for temperatures characteristic for novae explosions $T_9 = 0.2$–1.0, to average energies of 25–130 keV. These are very small energies for the nuclear laboratory and is difficult to conduct experiments at such energies. Furthermore, in the case of reactions between charged partners, there is a Coulomb barrier which introduces large hindrances for low energies. The cross sections become very small, due to small penetrabilities through the barrier. It is customary for such reactions to introduce the astrophysical S-factor instead of the cross section:

$$\sigma(E) = \frac{S(E)}{E}\exp(-2\pi\eta) \tag{2}$$

where $\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$ is the Sommerfeld parameter and v is the relative velocity. The factor $\exp(-2\pi\eta)$ accounts for the penetrability through barrier (and is exact for an *s*-wave penetrability all the way down to $r = 0$) and, therefore, the astrophysical factor $S(E)$ incorporates the nuclear effects. What matters in the end in stars is the reaction rate, that is, the number of reaction events: $1 + 2 \to 3 + 4$ per unit of time

$$R_{12} = \frac{n_1 n_2}{1+\delta_{12}}\langle\sigma v\rangle_{12} \tag{3}$$

where n_1 and n_2 are the number densities of particles of type 1 and 2, and $\langle \sigma v \rangle$ is the average of cross section times velocity over the Maxwell-Boltzmann distribution. The definition of the reaction rates is valid for any type of reactions, not only for those between charged particles. In the case of charged particle reactions with astrophysical *S*-factors varying smoothly (no resonances), the product of the cross sections (very small at small energies and exponentially increasing) with the distribution probability (exponentially decreasing for large energies) leads to significant contributions from a relatively narrow region, called Gamow peak (Fig. 1), with a maximum at

$$E_0 = \left[\frac{\pi e^2 Z_1 Z_2 kT}{\hbar c} \left(\frac{\mu c^2}{2} \right)^{1/2} \right]^{2/3} = 1.22 \mu^{1/3} \left(Z_1 Z_2 T_6 \right)^{2/3} \text{ keV} \tag{4}$$

and the width

$$\Delta E_0 = 4 \left(\frac{E_0 kT}{3} \right)^{1/2} = 0.749 \left(Z_1^2 Z_2^2 \mu T_6^5 \right)^{1/6} \text{ keV} \tag{5}$$

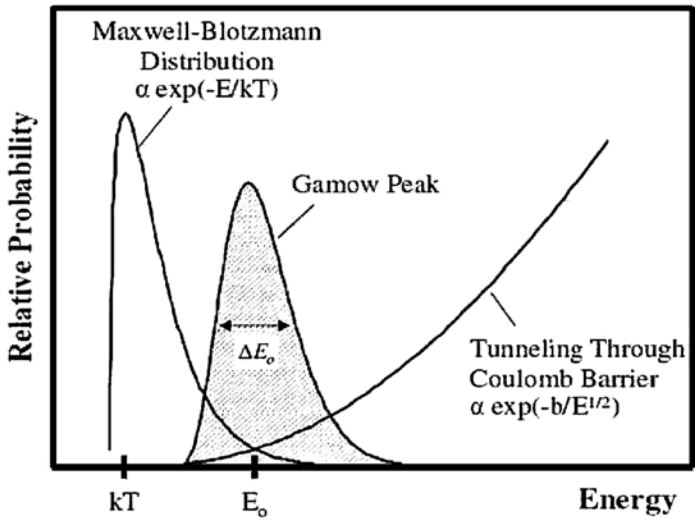

Fig. 1 – Schematic presentation of the Gamow peak region for reactions with charged particle at stelar temperatures.

In the formulae above Z_1, Z_2, and μ are the atomic numbers and the reduced mass of the partners in the entrance channel, respectively. In cases where we have narrow resonances in our processes, corresponding into abrupt variations in the excitation functions [and $S(E)$], extra contributions appear in the reaction rate (3) for each resonance *i*

$$\langle \sigma v \rangle_{res} = \left(\frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \sum_i \left[\frac{(2J+1)}{(2J_1+1)(2J_2+1)} \frac{\Gamma_{in}\Gamma_{out}}{\Gamma_{tot}} \exp\left(-\frac{E_{res}}{kT} \right) \right]_i \tag{6}$$

$$= \left(\frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \sum_i \left[(\omega\gamma)_{res} \exp\left(-\frac{E_{res}}{kT} \right) \right]_i \tag{7}$$

with Γ_{in}, Γ_{out}, Γ_{tot} the partial and total widths of the resonance, E_{res} the position of the resonance and J the spins of the resonance and of the incoming particles. $(\omega\gamma)_{res}$ is called the resonance strength. It is clear that what we need to know here for the evaluation of the contribution of narrow resonances to the reaction rate are the positions of the resonances and their resonant strengths $\omega\gamma$. To obtain the formulae above is a simple but useful exercise, I will not insist here. For more details and definitions consult a book on nuclear astrophysics, like the very popular one of Rolfs and Rodney [4], or the original article of Burbidge *et al.* [5]. It is useful, though, to evaluate the position of the Gamow peak in a few cases. In the case of Sun, the Gamow peaks are around $E_0 \approx 20$ keV (depending slightly of the reaction considered). The corresponding cross sections are of the order of fb (femtobarn) and below. In the case of He burning, in one of the most important reactions in nuclear astrophysics $^{12}C(\alpha, \gamma)^{16}O$ at $T_9 = 0.2$, the Gamow peak is located at $E_0 \approx 320$ keV and has a width $\Delta E_0 \approx 170$ keV. I want to stress here that in both cases the direct measurements cannot go as far down in energy as we would need, and the nuclear astrophysicists have to make difficult measurements at low energies, and then extrapolate to the region of the Gamow peak. The extrapolations are in themselves a problem, they have to be theory guided, and in many cases, the two examples above included, they are complicated and uncertain.

A. THE $^7Be(p, \gamma)^8B$ REACTION IN THE SUN

I shall say a few words to introduce this reaction which I choose to use as an example for three of the indirect methods with radioactive beams treated in the next section. I choose it because this is one of the most important reactions in nuclear astrophysics, with implications far beyond our understanding of H burning in our Sun, extending into the understanding of neutrino physics, and therefore it was studied in many laboratories and through many types of experiments.

The search for solar neutrinos and later for a solution of the solar neutrino problem was a major and extremely fruitful quest of the last few decades (see a good review in Ref. [6]). It involved major experimental and theoretical efforts and lead to major breakthroughs, such as the confirmation of the neutrino oscillations ([7–10] and the references therein). We knew for sometime that only nuclear reactions could be the source of Sun's energy and that those reactions have to take place deep inside the core of the Sun, at temperatures much larger than those of the corona, the region from which the light comes to us. The energy is produced in the pp chains and in the CNO cycle, both with the same result of transforming 4 protons into a ^{4}He nucleus, 2 positrons, 2 neutrinos and energy. The neutrinos produced in the nuclear reactions are the only signal that can carry information about those reactions from the interior of the Sun all the way to our terrestrial detectors. Essentially, only when solar neutrinos were detected by Davis [7], we could say we had experimental evidence that nuclear reactions are indeed fueling the Sun, and probably the other stars too. Solar neutrinos are produced at different stages in the pp burning processes. In one of them, the pp-III chain, which only accounts for about 0.1% of the energy balance, ^{7}Be nuclei are produced. Further, they capture another proton through the ^{7}Be(p, γ)^{8}B reaction. ^{8}B is not stable, it beta-decays into a short lived ^{8}Be, a positron and a neutrino that can have a maximum energy of 15 MeV, the largest energy of all neutrinos produced in solar processes. However, most of the neutrinos detected by SuperKamiokande and SNO detectors are ^{8}B neutrinos because of the selectivity of the detection processes used, and therefore, for reliable calculations of the neutrino flux in the standard solar model [10] we need good data for the reaction rates involved in the pp-III chain, in particular for the ^{7}Be producing reaction ^{3}He(α, γ)^{7}Be and for the radiative proton capture on it ^{7}Be(p, γ)^{8}B. The uncertainties in the cross sections of these two reactions are dominating the uncertainties in the calculations of neutrino production in Sun. The magnitude of the latter's reaction cross section is measured by what we call the S_{17} astrophysical factor. The precise determination of S_{17} was and is, therefore, the subject of a large number of experimental and theoretical efforts ([11–19], to name only a few of the most recent), summarized in many recent publications [20–22]. In spite of these efforts there is no clear consensus on the value of $S_{17}(0)$ at the desired 5% precision, with apparent discrepancies between values given by some direct measurements and some from indirect methods, like Coulomb dissociation [17, 20, 23], (^{7}Be, ^{8}B) proton transfer reactions [13, 26] and breakup of ^{8}B [15, 27–29]. Consequently, several new experiments are under way or planned.

The average value obtained by Cyburt *et al.* [22] $S_{17}(0) = 20.8 \pm 0.6(stat) \pm$ $\pm\, 1.0(syst)$ eV b using all radiative capture data (direct measurements) is dominated by a single measurement that claims a very good precision [21] $S_{17}(0) = 22.1 \pm 0.6(stat) \pm 0.6(theor)$ eV b.

III. INDIRECT METHODS IN NUCLEAR ASTROPHYSICS

The principle behind the use of indirect methods is simple: we (1) do experiments at higher energies (tens of MeV/u or higher), typically above the Coulomb barrier, where cross sections are larger, in order to obtain nuclear information that we consequently (2) transform in reaction cross sections and reaction rates for astrophysically relevant energies (see Fig. 2). I want to stress here that in both steps above, calculations are needed, and therefore good knowledge of the dynamics of the processes involved is crucial. Typically more information from other experiments is needed or valuable. Another point I want to stress is that it is important to choose, and therefore seek at step (1), the nuclear information (typically nuclear structure information) that is the most relevant in evaluating the reaction rates sought at step (2). For this see the discussion about ANCs *vs*. spectroscopic factors in Sect. 3.B and that about the spin of the g.s. of ^{23}Al in Sect. 3.D.

There are only a few indirect methods applied in nuclear astrophysics:

a) Coulomb dissociation
b) transfer reactions (the ANC method)
c) breakup at intermediate energies
d) Trojan horse method
e) other spectroscopic methods, in particular the study of resonances.

I will discuss each of them in what follows, except d), the Trojan horse method, which I personally only brushed with briefly (and because of time constrains). See Refs. [30, 31] for this subject. Methods a), b) and c) will be presented in connection with the study of the ^{7}Be(p, γ)^{8}B reaction, crucial for the solar neutrino production, and I will present more extensively examples for cases b), c) and e).

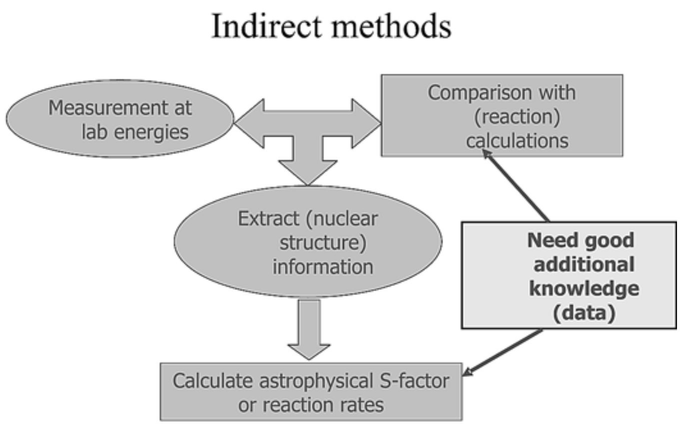

Fig. 2 – Scheme of the logic of indirect methods used in nuclear astrophysics.

A. THE COULOMB DISSOCIATION

The Coulomb dissociation is a method specifically introduced for nuclear astrophysics two decades ago [32, 33], and the Carpathian summer school in

Predeal/Poiana Brasov was one of the first places where prof. Rebel talked about it. It can be used to determine cross sections (or equivalently, astrophysical S-factors) for radiative capture reactions with charged particles (protons or alphas). Schematically, it works as sketched in Fig. 3! Instead of studying the radiative capture reaction $B(p, \gamma)A$ at a definite center-of-mass energy E_p, process in which a gamma-ray of energy $E_\gamma = E_p + S_p$ is emitted (S_p =binding energy of the proton in nucleus A) (top part of Fig. 3), we could measure the inverse process: photodissociation (bottom). There a photon of energy E_γ interacts with nucleus A producing the dissociation $A + \gamma \rightarrow B + p$, in which a proton-core system of energy $E_p = E_\gamma - S_p$ results. Then the Fermi golden rule of detailed equilibrium can be used to relate the cross sections of the two processes. Baur, Bertulani and Rebel proposed to replace the real photons needed in photodissociation with virtual photons. A fast moving projectile A in the strong Coulomb field of a high Z target like Pb senses a field of virtual photons that induce the dissociation of the projectile $A \rightarrow B + p$. The resulting cross section for Coulomb dissociation is a product between the photodissociation cross section and the number of virtual photons of the particular multipolarity and energy needed:

$$\frac{d^2\sigma}{dE_\gamma d\Omega}(E_\gamma, \theta) = \frac{1}{E_\gamma}\left[\frac{dN(E1, E_\gamma)}{d\Omega}\sigma_{E1}^{photo}(E_\gamma) + \frac{dN(E2, E_\gamma)}{d\Omega}\sigma_{E2}^{photo}(E_\gamma)\right] \tag{8}$$

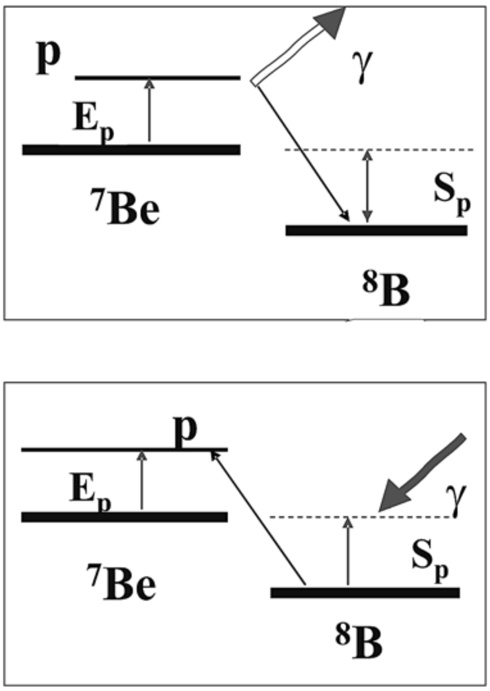

Fig. 3 – Radiative capture (top) and its inverse reaction, photodissociation (bottom). The photons are virtual in Coulomb dissociation.

The photodissociation cross section is directly related to the radiative capture cross section sought in nuclear astrophysics. Measuring the cross section for Coulomb dissociation as a function of the relative energy p-core E_p we can extract the energy dependence of the astrophysical factor $S(E_p)$. In reality problems may arise from the need of relatively large projectile incident energies to produce enough virtual protons of the large E_γ energy needed to produce photodissociation. Another problem stems from the fact that different multipoles contribute in different proportions in Coulomb dissociation and in radiative capture (see Eq. above). In Coulomb dissociation, at the large energies involved a mixing of E1 and E2 multipolarities occurs, while in radiative capture at low energies only E1 contributes. Therefore a disentangling of different multipole contributions from angular distribution measurements in Coulomb dissociation is needed before translating the results into astrophysical S-factors for radiative capture. That is experimentally very demanding. Also, it is difficult, if not impossible, to separate the contribution of the nuclear field from that of the Coulomb field in dissociation at large energies. This is done requiring that dissociation happens at large impact parameters, which translates into measurements very close to zero degrees, experimentally a difficult task. However a number of very good Coulomb experiments have been done to obtain astrophysical data, too many to cite them all, and the method is considered rather well established. One important conceptual advantage of the method is that from Coulomb dissociation the energy dependence $S(E)$ can be experimentally extracted (E = relative p-core energy). Currently the method is considered very appropriate for use with proton rich radioactive beams at intermediate energies obtained through projectile fragmentation. Notable contributions were brought by the Coulomb dissociation studies of $^8B \to {}^7Be + p$ to the determination of the S_{17} factor for solar neutrinos [17, 20, 23]. Initially somewhat lower values were obtained from Coulomb dissociation data than from direct measurements $S_{17}(0) = 18.6 \pm 0.4(exp) \pm 1.1(syst)$ eV b [20], a latter re-analysis of the GSI2 set of data gets even closer [24].

B. THE ANC METHOD

In the past years at the Cyclotron Institute at Texas A&M University we used the proton transfer reactions $^{10}B(^7Be, ^8B)^9Be$ [13] and $^{14}N(^7Be, ^8B)^{13}C$ [25, 26] and what has become known as the Asymptotic Normalization Coefficient (ANC) method [35] to determine the amplitude of the tail of the overlap integral of the ground state wave function of ^8B onto the two-body channel ^7Be + p. It is known for long time [36] that this normalization of the wave function of the last proton at large distances determines entirely the astrophysical S-factors for charged particle radiative capture (Fig. 4). The method consists in determining this normalization constant from transfer reactions (Fig. 5) at 10–15 MeV/u, which are also peripheral, but happen at smaller distances core-proton and, therefore, have much larger cross sections. Let us take a few moments to describe the basic ideas of the approach. For the case of radiative proton capture $B(p, \gamma)A$ at low energies (or the radiative capture of any charged particle for that matter) (Fig. 4), the process is very peripheral. This can be easily understood by the presence of the Coulomb barrier. Classically, for energies bellow the barrier, there will be a turning point for the protons approaching nucleus B (^{7}Be core in this example) and no capture will be possible at all. Quantum mechanics allows the penetration through the barrier, but the further in the penetration, the smaller the flux penetrating. The cross section is given by the square of a matrix element of the electromagnetic operator $O(EL)$ between the penetrating wave $\chi^{(+)}$ in the input channel (unbound proton of energy E_p in the Coulomb field), and the wave function Φ of the bound final state (binding energy S_p of nucleus A (^{8}B here):

$$\sigma_{rc}(E_p) = \lambda \left| \left\langle \Phi(A) \middle| O(EL) \middle| \Phi(B)\chi^{(+)} \right\rangle \right|^2 \tag{9}$$

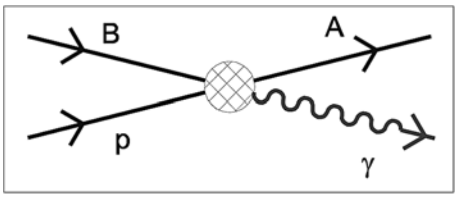

Fig. 4 – The reaction vertex in radiative proton capture reactions.

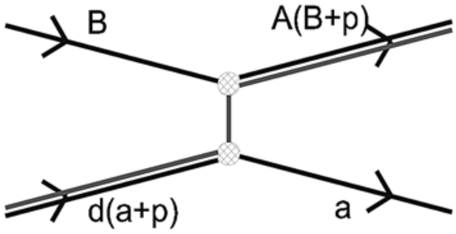

Fig. 5 – The reaction vertices in the proton transfer reaction $B(d, a)A$.

The electromagnetic operator does not affect the structure of core B and one can integrate in Eq. 9 over its intrinsic coordinates, which leads to the overlap integral $I(r) \equiv \langle \Phi(A)\Phi(B) \rangle$. At large distances the overlap integral is a solution of the Schroedinger equation of a proton of negative energy $-S_p$ in Coulomb field and has a well known shape

$$I_{Bp}^{A}(r) = C_{nlj}^{A} \frac{W_{-\eta, l+1/2}(2\kappa r)}{r} \tag{10}$$

where W is the Whittaker function, $\kappa = \left(2\mu S_p\right)^{1/2}$ is the wave number for the bound proton, nlj are the usual quantum numbers for single particle wave functions and C_{nlj}^{A} is the asymptotic normalization coefficient (ANC) of the ground state of A projected into the nlj channel (further I will use the same name for the square of this number). This ANC depends of the structure of the nucleus A.

The cross section for the radiative capture (Eq. 9) will only contain contributions from large distances (due to the penetration through barrier) where we can approximate the overlap integral with its asymptotic form. It becomes now

$$\sigma_{rc}(E_p) = \lambda \left| \left\langle I(r) \middle| O(EL) \middle| \chi^{(+)} \right\rangle \right|^2 = \\ = \lambda \left| \left\langle C \frac{W(r)}{r}) \middle| O(EL) \middle| \chi^{(+)} \right\rangle \right|^2 = C^2 \cdot w(E_p) \tag{11}$$

where $w(E_p)$ is a function which does not depend on the particular structure of the bound state of A (g.s. of ^{8}B), and can be calculated. Therefore the capture reaction cross section is completely determined by the ANC. In the equations above, for the sake of simplicity I considered that only one single particle orbital *nlj* participates; typically this is not the case, and a summation over all participating orbitals is needed. If we are able to determine this ANC (these ANCs) from other experiments, we can evaluate the capture cross section at stellar energies. The condition is to use peripheral phenomena, that occur at distances where we can safely approximate the overlap integrals with the asymptotic forms in Eq. 10. The proton transfer reactions between heavy ions are such phenomena.

We can choose proton transfer reactions such that one of the vertices is the same vertex that appears in the proton radiative capture reaction (Fig. 5). Typically, from nucleon transfer reactions, spectroscopic factors $S^A_{n_A l_A j_A}$ are extracted. If the conditions are such that the reactions are peripheral, we can approximate the overlap integrals with the asymptotic forms (10) and we can extract the asymptotic normalizations $C^2_{n_A l_A j_A}$ by fitting the experimental angular distributions with calculated DWBA cross sections:

$$\frac{d\sigma}{d\Omega} = \sum S^A_{n_A l_A j_A} S^d_{n_d l_d j_d} \sigma_{DWBA}(\theta) = \sum C^2_{n_A l_A j_A} C^2_{n_d l_d j_d} \frac{\sigma_{DWBA}(\theta)}{b^2_{n_A l_A j_A} b^2_{n_d l_d j_d}} \tag{12}$$

The coefficients b_{nlj} are the ANC of the normalized single particle wave functions used in the DWBA calculations. Comparing the experimental results with DWBA calculations, we can determine the spectroscopic factor, or the ANC for the vertex we are interested in ($A \to B + p$), provided we have already the information for the other vertex ($d \to a + p$). The advantages of using transfer reactions at laboratory energies are explained schematically in Fig. 6. In figure a single particle bound wave function $1p_{3/2}$ is shown, together with a Whittaker function asymptotically matched to it. Radiative capture happens when the bound proton is far away from the center of the core as shown at the right of figure (around 50 fm for ^{8}B at solar energies), whereas proton transfer (^{7}Be, ^{8}B) at about 10 MeV/u is also peripheral (leftmost bell shaped curve centered at about $r = 5$ fm shows its form factor), where the overlap integral has still its asymptotic form, but samples it much closer to the center and there is a gain of several orders of magnitude in the cross section due to this fact alone. Another advantage is that experiments around 10 MeV/u are easier to perform in the existing and planned nuclear physics laboratories.

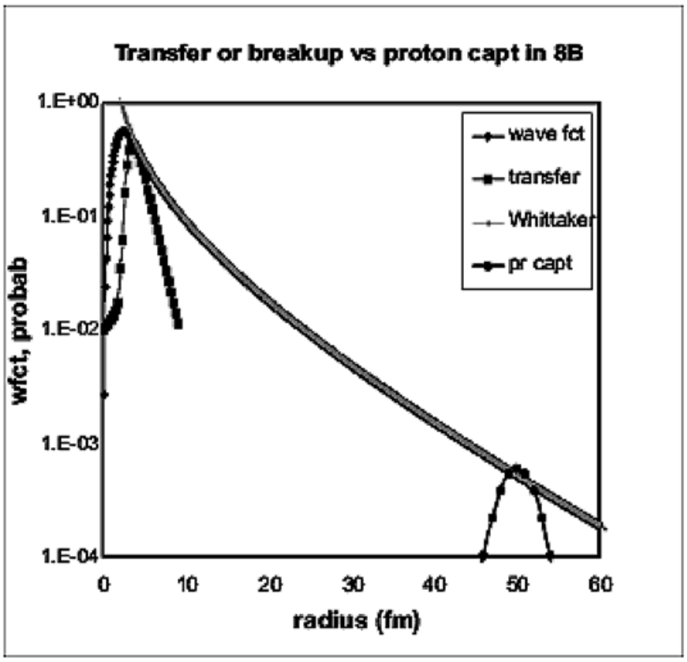

Fig. 6 – Advantages of use of transfer or breakup reactions at large laboratory energies, *vs.* direct proton capture measurements at very low energies. The overlap integral is sampled at different distances: at around 50 fm (capture), or just outside the Coulomb barrier (transfer and breakup) leading in orders of magnitude increase in the corresponding cross sections.

The main advantage of extracting the ANCs, versus the spectroscopic factors, is that the former are insensitive to the choice of the geometry of the nucleon binding potential (reflected in the single particle ANCs b_{nlj}^2) that we chose in the DWBA calculations (see Fig. 7). The ANC method was proved to reproduce within 9% the results from direct measurements in the case of the proton radioactive capture ^{16}O(p, γ)^{17}F [37] and was successfully used for a number of cases. The two (^{7}Be, ^{8}B) experiments cited above lead to mutually consistent ANC values. To analyze the transfer reaction d(^{7}Be, ^{8}B) a using DWBA calculations to extract the ANC, the optical model potentials (OMP) are

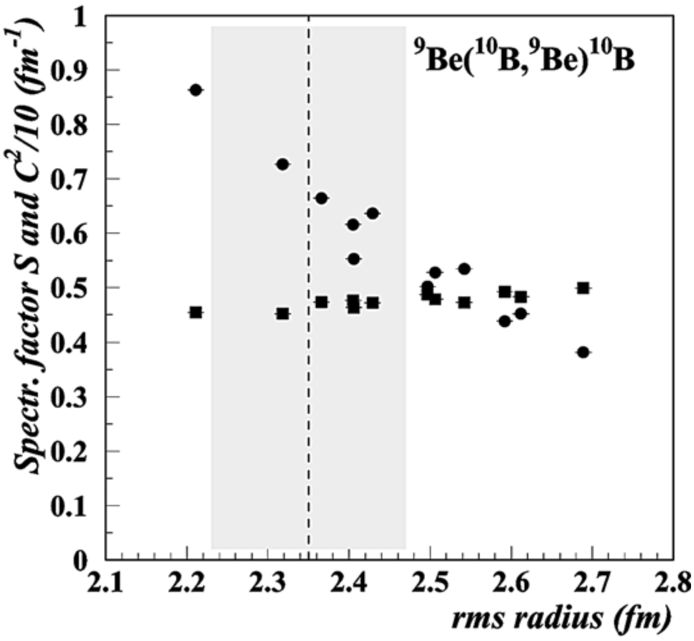

Fig. 7 – Spectroscopic factors (circles) and ANCs (squares) are extracted from experimental data using proton binding potentials of different geometries. The ANC values are insensitive to the geometry chosen. For each geometry (r_0, a) used, the rms radius of the ^{10}B charge distribution was calculated and is used as ordinate in the graph. The vertical bar shows the experimental radius with its error (shaded area).

needed in both the incoming ^{7}Be+^{10}B (or ^{14}N) channel and in the outgoing ^{8}B + ^{9}Be (or ^{13}C) channel. In both measurements the radioactive beam ^{7}Be at 12 MeV/u was obtained by bombarding a H_2 cryogenic gas target with ^{7}Li primary beam from the Texas A&M University superconducting cyclotron K500. Using the Momentum Achromat Recoil Separator (MARS), secondary beams of about 10^5 pps, high purity (above 99%) and sizes of about 4 mm diameter on target, were selected (Fig. 8). They bombarded secondary ^{10}B or melamine (composite $C_3N_6H_6$) targets, and 2 to 4 Si detector telescopes were positioned at angles covering angles 8 up to 30 deg to measure simultaneously elastic scattering and transfer reaction products. The measurement of the elastic scattering was made in order to get or check the optical potential used in the incoming channels in DWBA calculations for the proton transfer reactions (^{7}Be, ^{8}B). In the latter case, we also measured the elastic scattering of ^{8}B beam on a C target with the aim of checking directly, for the first time, the optical model we used in the outgoing channel ^{8}B+^{13}C. For more details, see references cited above. We also used the mixing ratio between the $1p_{1/2}$ and $1p_{3/2}$ determined in the mirror nucleus ^{8}Li from the one-neutron transfer experiment

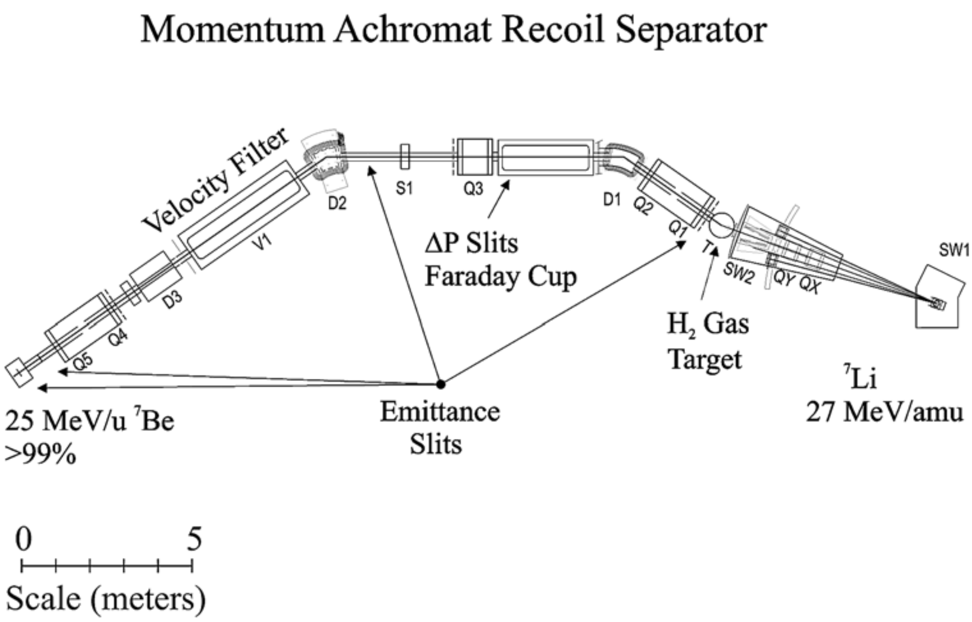

Fig. 8 – The production of radioactive beams in MARS.

(^{7}Li, ^{8}Li) [38]. The measurements allowed us to extract the Asymptotic Normalization Coefficient (ANC) of ^{8}B from the proton transfer (^{7}Be, ^{8}B) reaction: $C^2(^8\text{B}, p_{3/2}) = 0.418 \pm 0.040\ \text{fm}^{-1}$, resulting in $C^2_{tot}(^8\text{B}) = C^2_{p_{3/2}} + C^2_{p_{1/2}} = 0.471 \pm 0.044\ \text{fm}^{-1}$. This lead to an average value for the astrophysical S-factor for the ^{7}Be(p, γ)^{8}B reaction $S_{17}(0) = 18.2 \pm 1.7$ eV b, in reasonable agreement with the results of the other methods.

The astrophysical S-factors for the radiative proton capture reactions ^{11}C(p, γ)^{12}N, ^{13}N(p, γ)^{14}O and ^{12}N(p, γ)^{13}O were also obtained from the measurement of ANCs from proton transfer reactions at 12 MeV/u with radioactive beams separated with MARS: ^{14}N(^{11}C, ^{12}N)^{13}C [39], ^{14}N(^{13}N, ^{14}O)^{13}C [40] and ^{14}N(^{12}N, ^{13}O)^{13}C [41]. A large number of elastic scattering and transfer reaction studies with loosely bound but stable beams were made to establish and test the method, as well as to establish a successful semi-microscopic double folding procedure to predict OMP at around 10 MeV/u [42, 52].

C. BREAKUP OF LOOSELY BOUND NUCLEI AT INTERMEDIATE ENERGIES

A few years back we proposed to extract astrophysical S-factors from one-nucleon-removal (or breakup) reactions of loosely bound nuclei at intermediate energies or later [15, 34, 38]. The method is based on data showing that the structure of loosely bound nuclei is dominated by one or two nucleons orbiting a core. Consequently, we use the fact that their breakup is essentially a peripheral process, and therefore, the breakup cross-sections can give information about the wave function of the last nucleon at large distances from the core. More precisely, asymptotic normalization coefficients (ANCs) can be determined. We show that there exists a favorable kinematical window (about 30–150 MeV/u) in which breakup reactions are highly peripheral and are dominated by the external part of the wave function and, therefore, the ANC is the better quantity to be extracted. The approach offers an alternative and complementary technique to extracting ANCs from transfer reactions explained above, with the advantages that the cross sections are higher and that beams of much poorer quality and intensity are sufficient, making it very attractive for radioactive beams.

In the breakup of loosely bound nuclei at intermediate energies, a nucleus $B = (Ap)$, where B is a bound state of the core A and the nucleon p, is produced by fragmentation from a primary beam, separated and then used to bombard a secondary target. In measurements, the core A is detected, measuring its parallel and transverse momenta and eventually the gamma-rays emitted from its de-excitation (Fig. 9). Spectroscopic information can be extracted from these experiments, such as the orbital momentum of the relative motion of the nucleon and the contribution of different core states, typically comparing the measured

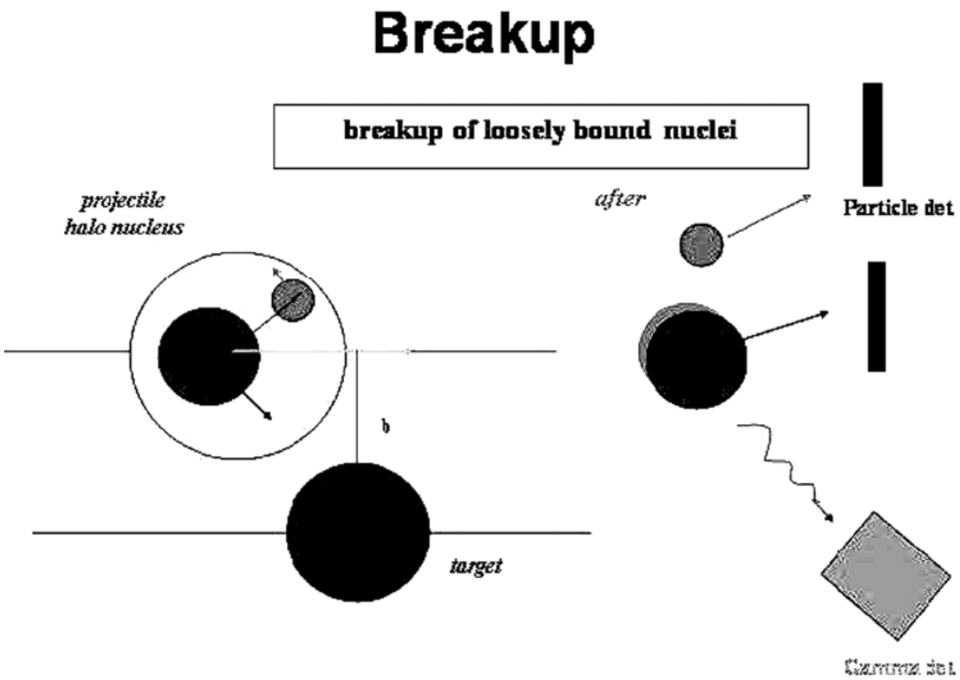

Fig. 9 – Breakup of loosely bound radioactive nuclei.

momentum distributions with those calculated with Glauber models. The integrated cross sections can be used to extract absolute spectroscopic factors [43] or the ANC [15]. The latter approach has, again, the advantage that it is independent of the geometry of the proton binding potential. Here first we use the well studied case of ^{8}B breakup as a benchmark to demonstrate the usefulness of the method and show the possibilities of the Glauber reaction model used (details of the model in refs. [44, 45]). The model calculations were extensively tested against existing experimental data: parallel and transversal momentum distributions on a wide range of targets, from Be to Pb, and energies. We have shown that existing experimental data at energies between 30 and 1000 MeV/nucleon [28, 46–49] on a range of light and heavy targets translate into consistent values of the ANC, which is then used to determine the astrophysical factor S_{17}. Two approaches were used in calculations. The first is a potential approach. To obtain the folded potentials needed in the *S*-matrix calculations we used the JLM effective nucleon-nucleon interaction [50], using the procedure and the renormalizations of ref. [52]. We applied this technique for energies below 285 MeV/nucleon only and on all targets. In a second approach, the Glauber model in the optical limit was used. Calculations were done using different effective nucleon-nucleon interactions with different ranges. No new parameters were adjusted. The contribution of the ^{7}Be core excitation was calculated for each target and at each energy using the data from an experiment which disentangle it [49], and corrected for in all cases. For details on the procedure see [29]. In Fig. 10 we show that from the widely varying breakup cross sections (upper panel) on all targets and at so different energies, we extract ANCs which are consistent with a constant value (lower panel). However, we see that a certain dependence on the used effective NN interaction exists, which may point to the limitations of our present knowledge of the effective nucleon-nucleon interactions. Taking the unweighted average of all 31 determinations we found an ANC $C^2_{tot}(breakup) = 0.483 \pm 0.050\ \text{fm}^{-1}$ (Fig. 10). The value is in agreement with that determined using the (^{7}Be, ^{8}B) proton transfer reactions at 12 MeV/u [13, 26]. The two values agree well, in spite of the differences in the energy ranges and in the reaction mechanisms involved. The ANC extracted leads to the astrophysical factor $S_{17}(0) = 18.7 \pm 1.9\ \text{eV·b}$ for the key reaction for solar neutrino production $^7\text{Be}(p,\gamma)^8\text{B}$. The uncertainties quoted are only the standard deviation of the individual values around the average, involving therefore the experimental and theoretical uncertainties. This 10% error bar is probably a good measure of the precision we can claim from the method at this point in time, due essentially to the uncertainties in the cross section calculations.

I want to underline that one of the very important contributions of the Coulomb dissociation, (^{7}Be, ^{8}B) proton transfer and breakup experiments described above is that they have given essentially the same result for S_{17} with a totally

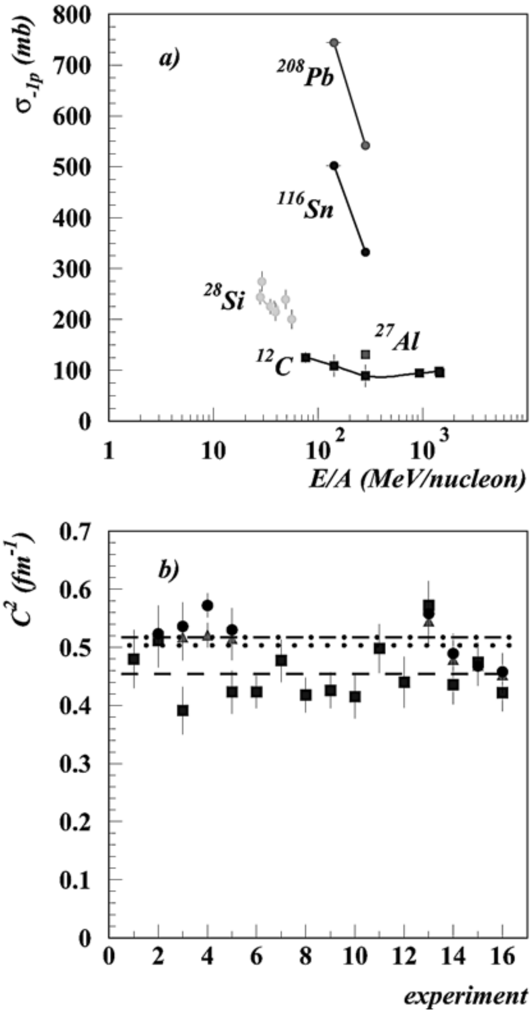

Fig. 10 – a) The one-proton-removal cross sections on C, Al, Sn and Pb targets, depending on energy. b) The ANCs determined from the breakup of ^{8}B at 28–1000 MeV/nucleon using the data above and various effective interactions: JLM (squares), "standard" (circles) and "Ray" (triangles). The dashed, dotted and dash-dotted lines are the averages of the three interactions above, in that order. See the list of experiments in Ref. [29].

different approach, and with totally different sources of errors or in-accuracies than the direct measurements, putting the nuclear physics part of the solar neutrino problem on a firm basis.

Further, a more recent experiment for the breakup of ^{23}Al is discussed to show that the method is particularly well adapted to rare isotope beams produced using fragmentation. As said before, space-based gamma-ray telescopes have the ability to detect γ-rays of cosmic origin. They have provided direct evidence that nucleosynthesis is an ongoing process in our Galaxy. Gamma-rays following the decay of long-lived isotopes like ^{26}Al ($t_{1/2} = 7.2 \times 10^5$ y), ^{44}Ti (60.0 y), ^{56}Ni (6.1 d), have been observed. Among the expected γ-ray emitters is also ^{22}Na ($t_{1/2} = 2.6$ y), thought to be produced in the thermonuclear runaway and in the high-temperature phase of so-called ONe novae (Oxygen-Neon novae) through the reaction chain ^{20}Ne(p, γ)^{21}Na(p, γ)^{22}Mg(βγ)^{22}Na – the so called NeNa cycle [4, 56–58]. Measurements, however, have not detected the 1.275 MeV gamma-ray from ^{22}Na. The origin of this discrepancy is not clear, but a poor knowledge of the reaction cross sections employed in the network calculations for the inset of the rp-process was presumed. In particular, it has been proposed that ^{22}Na itself or its precursor ^{22}Mg could be depleted by the radiative proton capture reactions ^{22}Na(p, γ)^{23}Mg [59] and ^{22}Mg(p, γ)^{23}Al [60], both leading to a serious reduction of the residual ^{22}Na abundance. The former is the prime candidate for this depletion, and uncertainties in the evaluation of its reaction rate are dominated by a poor knowledge of a the relevant resonances (positions and resonance strengths, see Eq. 6 and next subsection). For the second reaction a further complication appeared when some measurements of the reaction cross sections for $N = 10$ isotones and $Z = 13$ isotopes around 30 MeV/nucleon on a ^{12}C target found a remarkable enhancement for ^{23}Al, which led the authors to the conclusion that it is one of the rare proton halo nuclei [63]. This was explained with a presumed level inversion between the $2s_{1/2}$ and $1d_{5/2}$ orbitals. The inversion was further supported by several microscopic nuclear structure calculations [64, 65] To further complicate the things and confuse us, NNDC [51] gives $J^{\pi} = 3/2^{+}$. If the above mentioned inversion is correct, it will affect the radiative capture cross section much more strongly than any other uncertainties: we recalculated the astrophysical S-factor and the stellar reaction rate (Fig. 10b of our Ref. [66]) for the ^{22}Mg(p, γ)^{23}Al reaction and found an increase of 30 to 50 times over the current estimate of the rate for the temperature range $T_9 = 0.1$–0.3. We set to determine this spin by measuring the momentum distributions from the breakup of ^{23}Al in an experiment in GANIL (already successfully carried on at the time of typescript submission). The momentum distribution for the $2s_{1/2}$ case should be considerably narrower than the one for the $1d_{5/2}$ case. The data are still under analysis, but is already clear from the measurement of the gamma rays measured in coincidence with the ^{22}Mg core that a strong configuration mixing exists in the g.s. of ^{23}Al. Simultaneously we have addressed the problem of spin determination in experiments at the cyclotron of Texas A&M University, as described in the next subsection.

The main advantage of the breakup method is that it can be used with low quality RNBs from fragmentation, with intensities as low as a few pps (particles per second).

IV. OTHER SPECTROSCOPIC METHODS

There are a number of spectroscopic data that are either needed or useful in assessing astrophysically important reaction cross sections. I'll only discuss here two examples, both from the same study [67] of beta-decay of the proton rich nucleus ^{23}Al. Pure samples of ^{23}Al were obtained with a 48-MeV/nucleon ^{24}Mg beam bombarding a H_2 cryogenic gas target, 2.5 mg/cm^2 thick. Recoiling ^{23}Al nuclei from the p(^{24}Mg, ^{23}Al)2n reaction were separated in MARS 8. The secondary beam then passed through a 50-μm-thick Kapton foil into air, then through a plastic scintillator foil which counted the ions, through a set of Al attenuators, and finally stopped in the 76-μm-thick aluminized mylar tape of a fast tape-transport system. Our ^{23}Al beam was remarkably intense and pure (about 4000 pps and close to 100% purity) for these type of measurements of nuclei close to the proton dripline. Even though it was first produced some time ago, previous decay studies had to contend with very large backgrounds and could only study ^{23}Al through its beta-delayed proton-decay channel. For example, the previous best results [70] were obtained with about 20 pps on a background of 4000 pps. In our experiment the beam was pulsed in a simple scheme as shown in Fig. 11: 1 sec. irradiation time, 175 ms for the transport of the sample deposited on the tape to the location of detectors (about 1 m away from the place of irradiation), and 3.2 sec. measuring time. Beta singles and β-γ coincidences were measured with a plastic scintillator and a well efficiency calibrated HpGe detector. The measurements allowed us to determine the absolute branching ratios (the method itself is new and promoted in our group) and *ft*-values for transitions to final states in the daughter nucleus ^{23}Mg, including some with already known spins and parities. From the latter we unambiguously determined $J^\pi = 5/2^+$ for the ^{23}Al ground state. This contradicts the earlier suggestion that $J^\pi = 1/2^+$. The finding decreases the possibility of a simple explanation for the non-observation of the 1.275 MeV line from ^{22}Na decay in spectra taken by space-based gamma-ray telescopes. There is no evidence that its precursor, ^{22}Mg, can be depleted in ONe novae explosions by the reaction ^{22}Mg(p, γ)^{23}Al.

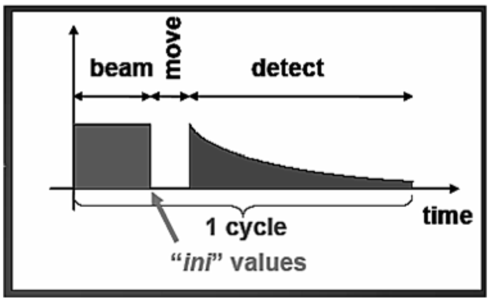

Fig. 11 – The time scheme of the irradiation and measurement.

Lifetime of ^{23}Al was determined with higher accuracy from multiscaling on selected gamma-rays. We have also found two states in ^{23}Mg with large beta-branching ratios at E^* = 7803(2) keV and 7787(2) keV, and identified them to be the isobaric analog state (IAS) of the ^{23}Al ground state and a $J^{\pi} = (7/2)^{+}$ state, which likely dominates the proton-decay spectrum. The IAS was unambiguously identified through the measurement of its log*ft*-value, which was found in agreement with the one for a pure $T = 3/2$ Fermi transition $\log ft = 3.31(3)$. Disentangling this doublet of states, not resolved in previously measured proton spectra from beta-delayed proton-decay of ^{23}Al, allowed us to solve an apparent puzzle [69] relative to a strong isospin mixing in daughter nucleus ^{23}Mg. Both these states are resonances contributing to the reaction ^{22}Na(p, γ)^{23}Mg which is suspected to contribute to the depletion of ^{22}Na in novae (Fig. 11). For the latter resonance at $E_{res} = 207(2)$ keV we find its resonance strength to be $\omega\gamma = 2.6(9)$ meV, making it the dominant contributor to the reaction rate at the temperatures of explosive H burning in ONe novae. This resonance strength determination, based on data from several types of indirect experiments (our measurement and those from refs. [69–71]) is in good agreement, but totally independent, from a determination from a difficult direct measurement ^{22}Na(p, γ)^{23}Mg involving a radioactive ^{22}Na target [68]. That alone, is remarkable and useful, as well.

V. CONCLUSIONS

A review of indirect measurements for nuclear astrophysics with emphasis on those usable with radioactive nuclear beams was made. The principles for the use of Coulomb dissociation, of proton transfer reactions and of the breakup reactions to obtain data for nuclear astrophysics are laid out. As examples for the methods, indirect measurements for the ^{7}Be(p, γ)^{8}B reaction are presented. The S_{17} values extracted from Coulomb dissociation, from proton transfer reactions (^{7}Be, ^{8}B) and from the breakup of ^{8}B are in agreement with all the values obtained from other indirect methods and with those from direct measurements. Our experience leads us to believe that one cannot expect a precision under 10% from this type of indirect determinations, but this is sufficient for most of the cases encountered in nuclear astrophysics. The results contributed to supplement the direct measurements for this very important reaction in the production of solar neutrinos and as benchmarks for the methods. That is very important for further measurements with radioactive beams in cases where direct measurements are impossible or very difficult. The importance of reliable calculations for the processes we study is emphasized.

In the last Section, I have shown how diverse spectroscopic data can be used to determine astrophysical reaction rates. The example used was that of the β–decay of the proton rich drip-line nucleus ^{23}Al. The determination of spectroscopic information for states that are becoming resonances in reactions at stellar energies is shown to become of particular importance. The example shown may not give enough credit to this kind of determinations, and the beta-decay chosen for exemplification is only one type of the many experiments one can use for this purpose.

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