EFFECTS OF THERMAL RADIATION ON MHD AND SLIP FLOW OVER A POROUS ROTATING DISK WITH VARIABLE PROPERTIES

EMMANUEL OSALUSI

International Centre for Island Technology (ICIT),
Institute of Petroleum Engineering, Heriot-Watt University,
Old Academy, Back Road, Stromness, Orkney Islands, Scotland, UK
osalusie@yahoo.com

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The combined hydromagnetic and slip flow of a steady, laminar conducting viscous fluid in the presence of thermal radiation due to an impulsively started rotating porous disk is studied taking into account the variable fluid properties (density, ρ, viscosity, μ, and thermal conductivity, κ). These fluid properties are taken to be dependent on temperature. The system of axisymmetric nonlinear partial differential equations governing the MHD steady flow and heat transfer are written in cylindrical polar coordinates and reduced to nonlinear ordinary differential equations by introducing suitable similarity parameters. The resulting steady equations are reduced to an initial valued problem and solved numerically using a shooting method. A parametric study of all parameters involved was conducted, and a representative set of results showing the effect of the magnetic field (M), the radiation parameter (N), slip factor (γ) the uniform suction parameter ($W < 0$) and the relative temperature difference parameter (ϵ) on velocities, temperature, skin-friction and Nusselt number are illustrated graphically. The numerical values of the wall shear stress and surface heat transfer are shown in tabular form, discussed quantitatively for parameter values of practical interest from physical point of view.

1. INTRODUCTION

The problem of hydrodynamic stability of flow due to a rotating disk has been the subject of study by several investigators since the pioneering work of von Karman [28]. Research into this type of flow has been spurred on by both theoretical imperatives as well as the practical applications of such flows, for example, in industrial machinery and lately, in computer disk drives, Herrero *et al.* [10]. The early study by von Karman has since been considerably extended starting with the work of Cochran [7] to include, *inter alia*, the effects of: (1) impulsively starting the flow from rest (Benton [4], Roger and Lance [23]), (2) an axial magnetic field applied to the fluid without Hall effects (El-Mistikawy *et al.* [8]), (3) an axial magnetic field with Hall effects (Attia and Aboul-Hassan

[3]) and (4) variable fluid properties (Maleque and Sattar [19]), (5) thermal radiation on MHD flow (Raptis *et al.* [22]) (6) variable properties with magnetic effect (Osalusi and Sibanda [21]).

The effects of variable properties on laminar boundary layers has been considered by, among others, Herwing [11] and Klemp [12]. Maleque and Sattar [19] have extended the consideration of the effects of variable fluid properties, namely the density, ρ, the viscosity μ and the thermal conductivity κ to flow due to a porous rotating disk. They found, among other things, that for fixed values of the suction parameter and Prandtl number, the momentum boundary layer increased considerably. Earlier work by Stuart [24] showed that the effect of suction is to thin the boundary layer by decreasing the radial and azimuthal components of the velocity while at the same time increasing the axial flow towards the disk at infinity. The recent study by Attia [2] considered the effect of temperature dependent viscosity on the flow and heat transfer along a uniformly heated impulsively rotating disk in a porous medium. In this study we extend the work of Malaque ans Satar [19] to include the effects of a magnetic field on flow due to a rotating disk in an electrically conducting fluid with temperature dependent density, viscosity and thermal conductivity.

Moreover, the thermal radiation of a gray fluid which is emitting and absorbing radiation in a non-scattering medium has been examined by Ali *et al.* [1], Ibrahim [15], Mansour [20], Hossain *et al.* [13, 14] and Elbashbeshy and Dimian [9]. The radiactive flows of an electrically conducting fluid with high temperature in the presence of a magnetic field are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications and other industrial areas [5, 26]. The MHD laminar convective flow of a conducting fluid with variable properties due to a porous rotating fluid which is assumed to be a gray emitting and absorbing radiation but non-scattering medium has not been studied.

A completely different extension of von Karman's one disk problem is the analysis of Sparrow *et al.* [6]. They considered the flow of a Newtonian fluid due to the rotation of a porous-surfaced disk and for that purpose replaced the conventional no-slip boundary conditions at the disk surface with a set of linear slip-flow conditions. A substantial reduction in torque then occurred as a result of surface slip. This problem was recently reconsidered by Miklavcic and Wang [27] who pointed out that the same slip-flow boundary conditions as those used by Sparrow *et al.* [6] also could be used for slightly rarefied gases or flow over grooved surfaces.

In all the above studies, the combined effects of slip and thermal radiation to the MHD flow and thermal fields over a rotating single disk has not been investigated and such is the motivation behind this study. In the following sections, the problem is formulated, analysed and discussed.

2. GOVERNING EQUATION

The description of the physical problem closely follows that of Osalusi and Sibanda [21]. We use a non-rotating cylindrical polar coordinate system, (r, φ, z) where z is the vertical axis in the cylindrical coordinates system with r and φ as the radial and tangential axes respectively. The homogeneous, electrically conducting fluid occupies the region $z > 0$ with the rotating disk placed at $z = 0$ and rotating with constant angular velocity Ω. The fluid velocity components are (u, v, w) in the directions of increasing (r, φ, z) respectively, the pressure is P, the density of the fluid is and T is the fluid temperature. The surface of the rotating disk is maintained at a uniform temperature T_{w}. Far away from the wall, the free stream is kept at a constant temperature T_∞ and at constant pressure P_∞. The external uniform magnetic field is applied perpendicular to the surface of the disk and has a constant magnetic flux density B_{o} which is assumed unchanging with a small magnetic Reynolds number $(Re_m \ll 1)$.

Following Jayaraj [17] (and more recently, Osalusi and Sibanda [21], Maleque and Sattar [19]), we assume that the dependency of the fluid properties, viscosity μ and thermal conductivity coefficients κ and density ρ are functions of temperature alone and obey the following laws;

$$\mu = \mu_\infty [T/T_\infty]^a, \qquad \kappa = \kappa_\infty [T/T_\infty]^b, \qquad \rho = \rho_\infty [T/T_\infty]^c, \tag{1}$$

where the a, b and c are arbitrary exponents, κ_∞ is a uniform thermal conductivity of heat, and μ_∞ is a uniform viscosity of the fluid. As in Osalusi and Sibanda [21] the fluid under consideration is a flue gas with $a = 0.7$, $b = 0.83$, and $c = -1.0$. The case $c = -1.0$ is that of an ideal gas. The physical model and geometrical coordinates are shown in Fig. 1. The equations governing the motion of the MHD laminar flow of the homogeneous fluid take the following form.

$$\frac{\partial}{\partial r}(\rho r u) + \frac{\partial}{\partial z}(\rho r w) = 0, \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z}\right) + \frac{\partial P}{\partial r} = \frac{\partial}{\partial r}\left(\mu\frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial r}\left(\mu\frac{u}{r}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) - \frac{\sigma B_o^2}{\rho}u, \tag{3}$$

$$\rho\left(u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z}\right) = \frac{\partial}{\partial r}\left(\mu\frac{\partial v}{\partial r}\right) + \frac{\partial}{\partial r}\left(\mu\frac{v}{r}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right) - \frac{\sigma B_o^2}{\rho}v, \tag{4}$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) + \frac{\partial P}{\partial z} = \frac{\partial}{\partial r}\left(\mu\frac{\partial w}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}(\mu w) + \frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right), \tag{5}$$

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = \frac{\partial}{\partial r}\left(\kappa\frac{\partial T}{\partial r}\right) + \frac{\kappa}{r}\frac{\partial T}{\partial r} + \frac{\partial}{\partial z}\left(\kappa\frac{\partial T}{\partial z}\right) - \frac{\partial q_r}{\partial z}, \tag{6}$$

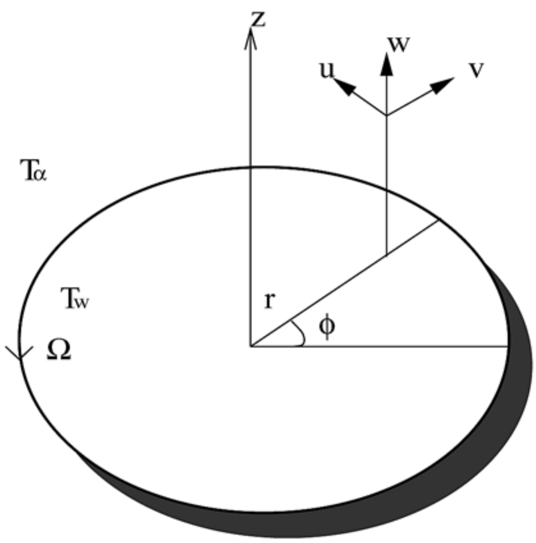

Fig. 1 – The flow configuration and the coordinate system.

where, σ is the electrical conductivity, c_p is the specific heat at constant pressure and q_r is the radiative heat flux.

When the mean free path of the fluid particles is comparable to the characteristic dimensions of the flow field domain, Navier-Stokes equations break down since the assumption of continuum media fails. In the range $0.1 < Kn < 10$ of Knudsen Number, the high order continuum equations, *e.g.* Burnett equations should be used. For the range of $0.1 < Kn < 10.001$, no-slip boundary conditions can not be used and should be replaced with the following expression (Ga-el-Hak [16]):

$$U_t = \frac{2-\psi}{\psi}\lambda\frac{\partial U_t}{\partial n} \tag{7}$$

where U_t is the target velocity, n is the normal direction to the wall, ψ is the target momentum accommodation coefficient and λ is the mean free path. For $Kn < 0.001$, the no-slip boundary condition is valid, therefore, the velocity at the surface is equal to zero. In this study the slip and the no-slip regimes of the Knudsen number that lies in the range $0.1 > Kn > 0$ is considered. By using equation (7), the boundary conditions are introduced [25] as follows:

$$\begin{aligned} &u = \frac{2-\xi}{\xi}\lambda\frac{\partial u}{\partial z}, \quad v = r\Omega + \frac{2-\xi}{\xi}\lambda\frac{\partial v}{\partial z} \quad W, \quad T = T_w, \quad at \quad z = 0 \\ &u \to 0, \quad v \to 0, \quad T \to T_w, \quad P \to P_\infty \quad as \quad z \to \infty. \end{aligned} \tag{8}$$

By using the Rosseland approximation for radiation for an optically thick layer [1] we can write

$$q_r = -\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial z}, \tag{9}$$

where σ^* is the Stefan-Boltzmann constant and κ^* is the mean absorption coefficient.

We assumed that the temperature differences within the flow are such that the term T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{10}$$

In view of (9) and (10) Eq. (6) reduces to

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = \frac{\partial}{\partial r}\left(\kappa\frac{\partial T}{\partial r}\right) + \frac{\kappa}{r}\frac{\partial T}{\partial r} + \frac{\partial}{\partial z}\left(\kappa\frac{\partial T}{\partial z}\right) + \frac{16\sigma^* T_\infty}{3k^*}\frac{\partial^2 T}{\partial z^2}, \tag{11}$$

3. SIMILARITY TRANSFORMATION

The solutions of the governing equations are obtained by introducing a dimensionless normal distance from the disk, $\eta = z(\Omega/\nu_\infty)^{1/2}$ along with the von-Karman transformations,

$$\begin{gathered} u = \Omega r F(\eta), \qquad v = \Omega r G(\eta), \qquad w = (\Omega\nu_\infty)^{\frac{1}{2}} H(\eta) \\ P - P_\infty = 2\mu_\infty \Omega p(\eta) \qquad \text{and} \qquad T - T_\infty = \Delta T \theta(\eta), \end{gathered} \tag{12}$$

where ν_∞ is a uniform kinematic viscosity of the fluid and $\Delta = T_w - T_\infty$. Substituting these transformations into equations (2)–(6) gives the nonlinear ordinary differential equations,

$$H' + 2F + cH\theta'(1+\epsilon\theta)^{-1}\epsilon = 0, \tag{13}$$

$$F'' + a\epsilon(1+\epsilon\theta)^{-1}\theta' F' - [F^2 - G^2 + HF' + MF(1+\epsilon\theta)^{-1}](1+\epsilon\theta)^{c-a} = 0, \tag{14}$$

$$G'' + a\epsilon G'(1+\epsilon\theta)^{-1} - [2FG + HG' + MG(1+\epsilon\theta)^{-1}](1+\epsilon\theta)^{c-a} = 0, \tag{15}$$

$$\theta''[3N(1+\epsilon\theta)^b + 4] + 3Nb\epsilon\theta'^2(1+\epsilon\theta)^{b-1} - 3NPrH\theta'(1+\epsilon\theta)^c = 0, \tag{16}$$

where, $Pr = \mu_\infty C_p/k_\infty$ is the Prandtl number, $M = \sigma B_0^2/\rho_\infty\Omega$ is the magnetic interaction parameter that represents the ratio of the magnetic force to the fluid inertial, $N = k^* k_\infty/4\sigma^* T_\infty^3$ is the radiation parameter and $\epsilon = \Delta T/T_\infty$ is the relative temperature difference parameter, which is positive for heated surface, negative for a cooled surface and zero for uniform properties. The prime symbol indicates derivative with respect to η. The equations differ from those in Osalusi and Sibanda [21] by way of the additional terms that involve the radiation parameter *N*. The transformed boundary conditions are given by;

$$F=\gamma F', \qquad G=1+\gamma G', \qquad H=W, \qquad \theta=1, \qquad \text{at} \qquad \eta=0 \tag{17}$$

$$F=G=\theta=p=0, \qquad \text{at} \qquad \eta\to\infty, \tag{18}$$

where $\gamma=[(2-\xi)\lambda\Omega^{1/2}]/\xi\nu^{1/2}$ is the slip factor, $W=w/\sqrt{\nu_\infty\Omega}$ represents a uniform suction ($W<0$) or injection ($W>0$) at the surface. The boundary conditions (18) imply that both the radial (*F*), the tangential (*G*), temperature and pressure vanish sufficiently far away from the rotating disk, whereas the axial velocity component (*H*) is anticipated to approach a yet unknown asymptotic limit for sufficiently large η-values.

The skin friction coefficients and the rate of heat transfer to the surface are given by the Newtonian formulas:

$$\tau_t=\left[\mu\left(\frac{\partial v}{\partial z}+\frac{1}{r}\frac{\partial w}{\partial \phi}\right)\right]_{z=0}=\mu_\infty(1+\epsilon)^a R_e^{\frac{1}{2}}\Omega G'(0),$$

and

$$\tau_r=\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\right]_{z=0}=\mu_\infty(1+\epsilon)^a R_e^{\frac{1}{2}}\Omega F'(0).$$

Hence the tangential and radial skin-frictions are respectively given by

$$(1+\epsilon)^{-a}R_e^{\frac{1}{2}}C_{f_t}=G'(0), \tag{19}$$

and

$$(1+\varepsilon)^{-a}R_e^{\frac{1}{2}}C_{f_r}=F'(0). \tag{20}$$

Fourier's law

$$q=-\left(\kappa\frac{\partial T}{\partial z}\right)_{z=0}=-\kappa_\infty\Delta T(1+\epsilon)^b\left(\frac{\Omega}{\nu_\infty}\right)^{\frac{1}{2}}\theta'(0), \tag{21}$$

is used to calculate the rate of heat transfer from the disk surface to the fluid. The Nusselt number *Nu* is obtained as

$$(1+\epsilon)^{-b}R_e^{\frac{1}{2}}Nu=-\theta'(0), \tag{22}$$

where $Re(=\Omega r^2/\nu_\infty)$ is the rotational Reynolds number.

4. METHOD OF SOLUTION

Equations (13)–(18) are solved numerically using a shooting method for different values of suction $W < 0$ and parameters Pr, ϵ, M, and N. To reduce the equations to first order equations we set $F = y_1$, $G = y_2$, $H = y_3$, $\theta = y_4$, $F' = y_5$, $G' = y_6$, $\theta' = y_7$ to get;

$$\begin{aligned}
&y'_1 = y_5, \qquad y_1(0) = \gamma s^{(5)},\\
&y'_2 = y_6, \qquad y_2(0) = 1 + \gamma s^{(6)},\\
&y'_3 = -2y_1 - cy_3y_7(1+\epsilon y_4)^{-1}\epsilon, \quad y_3(0) = W,\\
&y'_4 = y_7, \qquad y_7(0) = 1,\\
&y'_5 = -\epsilon a y_7 y_5(1+\varepsilon y_4)^{-1} + [y_1^2 - y_2^2 + y_5y_3 + My_1(1+\epsilon y_4)^{-1}](1+\epsilon y_4)^{c-a},\\
&y_5(0) = s^{(5)},\\
&y'_6 = -a\epsilon y_7 y_6(1+\epsilon y_4)^{-1} + [2y_1y_2 + y_3y_6 + My_2(1+\epsilon y_4)^{-1}](1+\epsilon y_4)^{c-a},\\
&y_6(0) = s^{(6)},\\
&y'_7 = 3N[y_3y_7Pr(1+\epsilon y_4)^c - b\epsilon y_7^2(1+\epsilon y_4)^{b-1}]/[3N(1+\epsilon y_4)^b + 4],\\
&y_7(0) = s^{(7)},
\end{aligned} \tag{23}$$

where $s^{(5)}$, $s^{(6)}$ and $s^{(7)}$ are determined such that $y_5(\infty) = 0$, $y_6(\infty) = 0$ and $y_7(\infty) = 0$. The essence of this method is to reduce the boundary value problem to an initial value problem and then use a shooting numerical techniques to guess $s^{(5)}$, $s^{(6)}$ and $s^{(7)}$ until the boundary conditions $y_5(\infty) = 0$, $y_6(\infty) = 0$ and $y_7(\infty) = 0$ are satisfied. The resulting differential equations are then be easily integrated using initial value solver, **lsode** and **fsolve** available in **GNU Octave**.

5. RESULTS AND DISCUSSIONS

Following Maleque and Sattar [19], the numerical solutions displayed in Tables 2–3 and Figs. 2–5 are relevant for a flue gas, that is, when $Pr = 0.64$. We have confined our analysis to the case when we have suction velocity only, that is, when $W < 0$.

For these calculations we took $Pr = 0.64$, which is the value of Prandtl number for a flue gas. We have confined our analysis to the case when we have suction velocity only, that is when $W < 0$.

Table 1 illustrates the effects of the parameters ϵ, M, γ and N on the shear stress $(F'(0), -G'(0))$ and the Nusselt Number $(-\theta'(0))$. We observe that the effect

Table 1

Variation of $F'(0)$, $-G'(0)$ and $-\theta'(0)$ at the disk surface with ϵ, M, γ and N parameters

ϵ	M	γ	N	$F'(0)$	$-G'(0)$	$-\theta'(0)$
0.1	0.1	0.1	1	0.146666507585938	1.305158143070613	1.108746805357471
0.2	0.1	0.1	1	0.155482091697311	1.224163337730904	1.078064017637734
0.3	0.1	0.	1 1	0.163284706379884	1.157744088104986	1.051338511165906
0.1	0.1	0.1	1	0.170240752026359	1.102398769584748	1.027779962727202
0.1	0.2	0.1	1	0.168055781768291	1.115001857897687	1.027748611028731
0.1	0.3	0.1	1	0.165920366357986	1.127476497359962	1.027718019146069
0.1	0.1	0.1	1	0.1702407520263591	1.1023987695847484	1.0277799627272017
0.1	0.1	0.2	1	0.1229786410220536	0.9935334620705638	1.0278869935116346
0.1	0.1	0.3	1	0.0918605745931315	0.9034659813321235	1.0278505264189997
0.1	0.1	0.1	1	0.170240752026359	1.102398769584748	1.027779962727202
0.1	0.1	0.1	2	0.170223899890198	1.102125662824946	1.037851936010839
0.1	0.1	0.1	3	0.170215640583929	1.101983677873937	1.042987524016908

of the increasing ϵ and M decreases the Nusselt number. Also the Nusselt number increases with an increase in the values of $0.1 \leq \epsilon \leq 0.3$ and $1 \leq N \leq 3$. Furthermore, the negative values of the wall temperature gradient, for all values of the dimensionless parameters, are indicative of the physical fact that the heat flow from the disk surface to ambient fluid. It can be seen that an increase in M decreases Nu while an increases in N for a fixed value of slip factor γ. Table 2 shows the numerical values of fluid temperature for different values of slip factor γ. We observe that θ decreases with an increase in the value of η. Further we also see that for fixed value of η, θ increases with γ. In Table 3, we notice that θ decreases with an increase in η. However, for a fixed η, an increase in N results

Table 2

Numerical values of the temperature obtained for several values of γ
for $\epsilon = 0.1$, $W = -1$, $M = 0.1$, $N = 1$ $Pr = 0.64$

η	$\theta(\eta)$ ($\gamma = 0.0$)	$\theta(\eta)$ ($\gamma = 5.0$)	$\theta(\eta)$ ($\gamma = 10.0$)
0.0	1.000000000000000	1.000000000000000	1.000000000000000
0.1	0.897527381722883	0.897659474221324	0.897670729830835
0.2	0.795585204017570	0.795847504935082	0.795869344475078
0.3	0.694186493463522	0.694569370533947	0.694600361222037
0.4	0.593347512410171	0.593830331438180	0.593868295967693
0.5	0.493085876130523	0.493635586957895	0.493677646354763
0.6	0.393419132442473	0.393990241134545	0.394032877147634
0.7	0.294363730980758	0.294899271726945	0.294938407539440
0.8	0.195934300091955	0.196367505841472	0.196398596585571
0.9	0.098143159407461	0.098399596543816	0.098417728687681
1.0	0.000999999999993	0.000999999997153	0.000999999998373

Table 3

Numerical values of the temperature obtained for several values of N for $\epsilon = 0.1$, $W = -1$, $M = 0.1$, $\gamma = 0.1$ $Pr = 0.64$

η	θ(η) (N = 0.0)	θ(η) (N = 5.0)	θ(η) (N = 10.0)
0.0	1.000000000000000	1.000000000000000	1.000000000000000
0.1	0.900100000000000	0.895609568725444	0.895166493054963
0.2	0.800200000000000	0.792100015296030	0.791290869238991
0.3	0.700300000000000	0.689511381007763	0.688420142851670
0.4	0.600400000000000	0.587888354259578	0.586606808761207
0.5	0.500500000000000	0.487277791034871	0.485906148295193
0.6	0.400600000000000	0.387726802774356	0.386374141917724
0.7	0.300700000000000	0.289281327791996	0.288065901429804
0.8	0.200800000000000	0.191985093110643	0.191034523731954
0.9	0.100900000000000	0.095878878190168	0.095330272266517
1.0	0.001000000000000	0.001000000000022	0.001000000000032

in a decrease in the value of θ. Fig. 2(a)–2(b) show the effects of M on the velocity (radial, tangential, axial). As it is seen in Fig. 2(a) the gradient in radial direction monotonically decreases with M and γ. Fig. 2(b) shows that as γ increases, the magnitude of the velocity gradient in tangential direction decreases. This is a consequences that the fluid cannot stick on the rotating disk; therefore, the slipping fluid decrease the surface skin friction. Since $G((\eta)$ inevitably has to change from $G(0)$ at the surface of the disk to zero at infinity, an increase in M, which has a thing effect on the circumferential boundary layer leads to an increase of the magnitude of $G'(0)$, for any value of γ.

Fig. 2(c) shows that the magnitude of the dimensionless temperature gradient at the surface monotonically decreases with M, however a careful look at the variation of $T'(0)$ with slip factor shows that for values of $0 \leq \gamma \leq 0.175$, $T'(0)$ increases with γ taking it maximum value, which is at: $\gamma = 0.075$, $T'(0) = -1.12969$ (for $M = 0$); $\gamma = 0.175$, $T'(0) = -1.12955$ (for $M = 5$); $\gamma = 0.175$, $T'(0) = -1.12935$ (for $M = 10$) and then decrease. We can infer that the maximum cooling of the rotating disk is reached at this value of the slip factor if the ambient fluid is colder than the rotating disk surface. Similarly solutions for the three velocity components (radial, tangential and axial) are presented in Figs. 3(a)–3(c) for some values of $0 \leq \gamma \leq 1$. The shear-driven motion (G) in the tangential direction in Fig. 3(b) is gradually reduced with decreasing values of γ, *i.e.* with an increasing amount of slip. The centrifugal force associated with this circular motion causes the outward radial flow (F) (Fig. 3(a)), which is correspondingly reduced with decreasing slip factor. The radial outflow F is compensated by an axial flow $-H$ (Fig. 3(c)) towards the rotating disk in accordance with mass conservation equation (13).

6. CONCLUSION

In this study, the radiation effect on steady MHD and slip flow over a rotating disk with variable properties is investigated. A solution for the velocity, temperature, shear stress and Nusselt number is obtained. The boundary layer equation has been solved numerically by the shooting method. The numerical results indicate that the radiation and slip factor have significant influences on velocity and temperature profiles, Nusselt number and shear stress.

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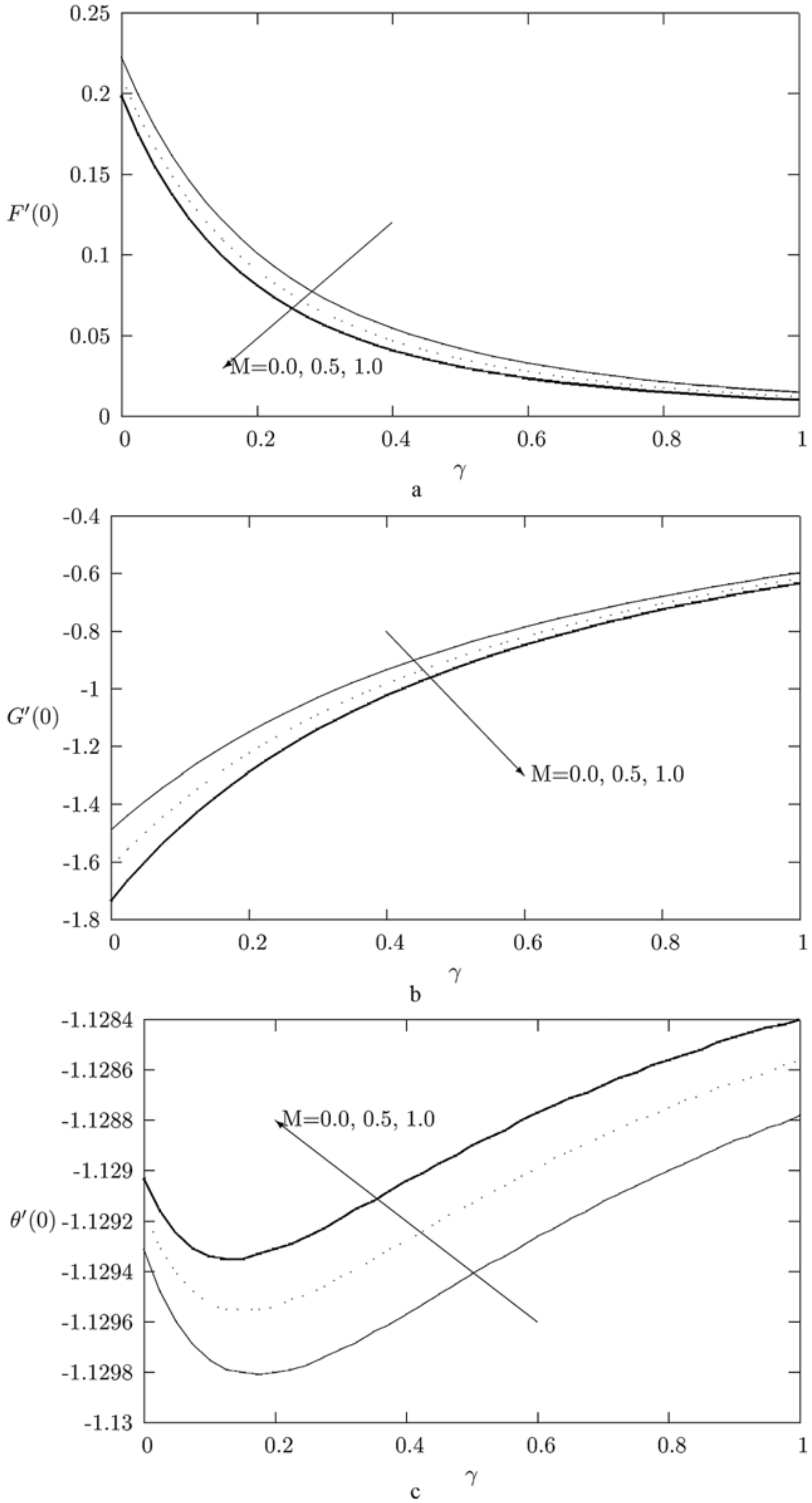

Fig. 2 – (a) Variation of $F'(0)$ with γ for several values of M: $\epsilon = 0.1$, $N = 1$, $Pr = 0.64$, $W = -1$; (b) Variation of $G'(0)$ with γ for several values of M: $\epsilon = 0.1$, $N = 1$, $Pr = 0.64$, $W = -1$; (c) Variation of $\theta'(0)$ with γ for several values of M: $\epsilon = 0.1$, $N = 1$, $Pr = 0.64$, $W = -1$.

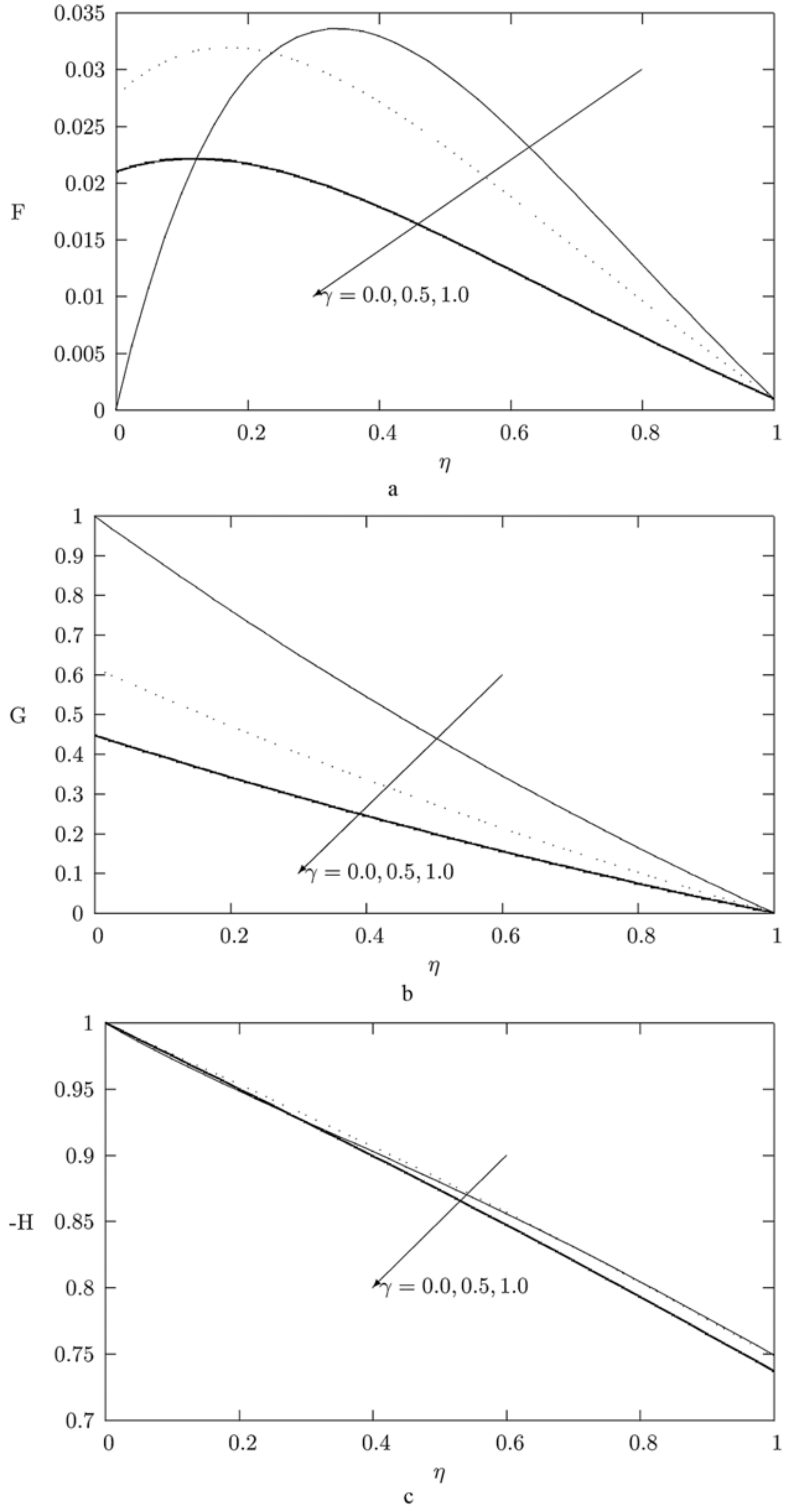

Fig. 3 – (a) Radial component of the velocity for several values of γ: $\epsilon = 0.1$, $N = 1$, $Pr = 0.64$, $M = 0.1$, $W = -1$; (b) Tangential component of the velocity for several values of γ: $\epsilon = 0.1$, $N = 1$, $Pr = 0.64$, $M = 0.1$, $W = -1$; (c) Axial component of the velocity for several values of γ: $\epsilon = 0.1$, $N = 1$, $Pr = 0.64$, $M = 0.1$, $W = -1$.