DISSIPATIVE DYNAMICS OF A SYSTEM OF FERMIONS

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In this paper, the dissipative dynamics of a system of matter particles, that from quantum point of view are Fermions, is described in the framework of a physical model. We show that dissipation consists in two-body correlations of the system with the environment particles. We obtain a quantum master equation with microscopic coefficients depending on the exactly known two-body potentials. We discuss this equation in comparison with other master equations, obtained on axiomatic grounds, or derived from a coupling with an environment of harmonic oscillators without altering the quantum conditions. Our master equation is in full accordance with the quantum-mechanical principles, with the detailed balance principle, and with other generally accepted conditions during the whole time-evolution: Pauli master equations for the diagonal elements of the density matrix, and damped Bloch-Feynman equations for the non-diagonal ones, that we call dynamical detailed balance. We show that the damping of a harmonic oscillator is not exponential as is generally accepted, but at lower energies, due to the decrease of the dipole moment, is slowing down. As applications, we study the super radiance of a semiconductor p-i-n structure with quantum dots.

1. INTRODUCTION

Dynamics of realistic matter-field system, as a system of electrons or nucleons interacting with the electromagnetic field, has always a substantial dissipative component caused by the interaction of the system with the environment. Dissipation, that in classical physics is described by friction forces, or by friction-diffusion coefficients for microscopic classical particles, in quantum mechanics is a difficult problem where specific conditions must be satisfied during the whole evolution of the system: the positivity of the probabilities, the reality of the observables, and the uncertainty relations. This field of investigation has a long history, a master equation satisfying these conditions being obtained only in 1976 by Lindblad, in the axiomatic framework of the semi-group theory. A physical interpretation of Lindblad's abstract coefficients has been found only 10 years later by Sandulescu an Scutaru.

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The dissipative quantum dynamics [1–3], that is essential in major application fields as quantum optics [4] and nuclear physics [5], is still raising very interesting problems [6–17]. Generally, now it is accepted that an *N*-level system is correctly described by a time-dependent semigroup of evolution operators [18–21], satisfying Lindblad's master equation [19]:

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{2\hbar}\sum_{n=1}^{N^2-1}([V_n\rho(t), V_n^+] + [V_n, \rho(t)V_n^+]). \tag{1}$$

This equation, depending on the dissipative operators

$$V_n = \sum_m a_{nm}s_m \tag{2}$$

as linear combinations of the system operators s_m, is valid only for a weak dissipative coupling. In comparison with other master equations taking into account a strong dissipative coupling [22, 23], it has the advantage of entirely preserving the quantum-mechanical properties of the density matrix (hermiticity, trace-class and positivity) during the whole evolution of the system. However, Eq. (1) is only a general form with $(N^2-1)^2$ free complex parameters for an *N*-level system that has only N^2-1 degrees of freedom. General conditions for describing the dynamics of a physical system in accordance with the detailed balance principle have only recently been derived [7]. In fact, Lindblad's master equation has been accepted as an appropriate physical tool only more than ten years after its publication, especially due to the connection of this axiomatic description with the previous phenomenological descriptions, realized by Sandulescu and Scutaru [24]. Thus, for a unidimensional system with the coordinate *x* and the momentum *p*, they defined the dissipative operators

$$V_n = a_n p + b_n x, \quad \text{with} \quad n = 1,2 \tag{3}$$

that lead to a quantum master equation

$$\begin{aligned}\frac{d}{dt}\rho(t) = &-\frac{i}{\hbar}[H, \rho(t)] - \frac{i\lambda}{2\hbar}([x, p\rho(t) + \rho(t)p] - [p, x\rho(t) + \rho(t)x]) - \\ &-\frac{D_{pp}}{\hbar^2}[x, [x, \rho(t)]] - \frac{D_{xx}}{\hbar^2}[p, [p, \rho(t)]] + \\ &+\frac{D_{px}}{\hbar^2}([x, [p, \rho(t)]] + [p, [x, \rho(t)]])\end{aligned} \tag{4}$$

with a friction coefficient

$$\lambda = \sum_{n=1}^{2}\frac{a_n^*b_n - a_nb_n^*}{2i}, \tag{5}$$

and three diffusion coefficients

$$D_{xx} = \frac{\hbar}{2}\sum_{n=1}^{2} a_n^* a_n, \quad D_{pp} = \frac{\hbar}{2}\sum_{n=1}^{2} b_n^* b_n, \quad D_{px} = -\frac{\hbar}{2}\sum_{n=1}^{2} \frac{a_n^* b_n + a_n b_n^*}{2}. \tag{6}$$

From these relations, between Lindblad's axiomatic coefficients a_n, b_n and the phenomenological coefficients λ, D_{xx}, D_{pp}, D_{px}, fundamental constraints for these coefficients are obtained:

$$D_{pp} > 0, \quad D_{xx} > 0, \quad D_{pp}D_{xx} - D_{px}^2 > \frac{\hbar^2\lambda^2}{4}. \tag{7}$$

It is remarkable that Eq. (4), based only on the condition that the evolution operators of the system form a time-dependent semigroup, satisfies also Heisenberg's uncertainty relation during the whole evolution of the system [24]. It is suggested that any values of the friction/diffusion coefficients satisfying the fundamental constraints are in principle allowed. Some interesting effects of quantum optics [25] and nuclear physics [26–28], predicted in this framework, have experimental evidence.

Considering an equilibrium asymptotic solution according to Boltzmann's distribution, for a harmonic oscillator with the frequency ω_0, this equation takes a form

$$\begin{aligned}\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] - \frac{\lambda}{2\hbar}\Big\{ & i([x, p\rho(t) + \rho(t)p] - [p, x\rho(t) + \rho(t)x]) + \\ & + \coth\left(\frac{\hbar\omega_0}{2T}\right)\left(\frac{1}{M\omega_0}[p,[p,\rho(t)]] + M\omega_0[x,[x,\rho(t)]]\right)\Big\},\end{aligned} \tag{8}$$

depending only on two parameters: the decay rate λ and temperature T. It is also remarkable that this equation, derived only from an asymptotic condition, satisfies also the detailed balance condition during the whole evolution of the system. Really, from (8), one obtains equations of matrix elements

$$\begin{aligned}\frac{d}{dt}\rho_{mn}(t) = -i(m-n)\omega_0\rho_{mn}(t) - \lambda\Big\{ & \left[(m+n+1)\coth\left(\frac{\hbar\omega_0}{2T}\right) - 1\right]\rho_{mn}(t) - \\ & -\sqrt{(m+1)(n+1)}\left[\coth\left(\frac{\hbar\omega_0}{2T}\right) + 1\right]\rho_{m+1,n+1}(t) - \\ & -\sqrt{mn}\left[\coth\left(\frac{\hbar\omega_0}{2T}\right) - 1\right]\rho_{m-1,n-1}(t)\Big\}\end{aligned} \tag{9}$$

that, for the diagonal ones, take the form of the Pauli classical master equation

$$\frac{d}{dt}\rho_{nn}(t)=\lambda\left\{(n+1)\left[\left(\coth\frac{\hbar\omega_0}{2T}+1\right)\rho_{n+1,n+1}(t)-\left(\coth\frac{\hbar\omega_0}{2T}-1\right)\rho_{nn}(t)\right]+\right.$$
$$\left.+n\left[\left(\coth\frac{\hbar\omega_0}{2T}-1\right)\rho_{n-1,n-1}(t)-\left(\coth\frac{\hbar\omega_0}{2T}+1\right)\rho_{nn}(t)\right]\right\} \tag{10}$$

with an asymptotic solution corresponding to Boltzmann's distribution

$$\frac{\rho_{n+1,n+1}(\infty)}{\rho_{nn}(\infty)}=\frac{\rho_{nn}(\infty)}{\rho_{n-1,n-1}(\infty)}=e^{-\frac{\hbar\omega_0}{T}}. \tag{11}$$

More than that, according to this equation, the population variation of an arbitrary level n is the result of the population transitions only from the two neighboring levels $n+1$, $n-1$. This corresponds to a dipole coupling of the harmonic oscillator that has non-zero dipole moments only between two successive levels. Really, the master equation (8) has been re-obtained for a harmonic oscillator coupled with the electromagnetic field in an independent oscillator model [29], that corresponds to an electric-dipole interaction, in this model the field variation with the oscillator coordinate not being taken into account.

However, although the diagonal elements satisfy equations in agreement with the principle of detailed balance in a sense recently discussed [7–9], the equations (9) of the non-diagonal matrix elements describe non-physical couplings of a transition $m\leftrightarrow n$ with the neighboring transitions $m-1\leftrightarrow n-1$ and $m+1\leftrightarrow n+1$. The possibility of such couplings is discussed in [30], as representing a coherence transfer between equidistant levels - in this case, all the transitions between equidistant levels should be coupled. This is not the case here, where the couplings of the transitions between the neighboring levels appear merely by using Lindblad's master equation with only two operators x and p – these couplings are present for an arbitrary unidimensional potential [24, 31]: the derivation of Eq. (4) is exclusively based on general combinations V_n of the system operators, without any additional assumption about the potential. In fact, one can not expect that the quantum dynamics of an N-level system be described very precisely with only two operators x and p, even though this system has the simpler form of a harmonic oscillator. This description with only two operators has been used to obtain a master equation in agreement with the quantum-mechanical principles, without increasing too much the number of the free parameters introduced through Lindblad's axiomatic formalism [24].

In this paper we consider the general case of an arbitrary system of Fermions coupled to a complex dissipative environment including other Fermions [32], Bosons [33], and the free electromagnetic field [34]. We obtain a general quantum master equation with explicit microscopic coefficients depending on the physical characteristics of the system: two-body potentials, densities of states, occupation probabilities of these, temperature. This equation

is in full agreement with the quantum-mechanical principles, with the detailed balance principle, and with a dynamical principle that we call "dynamical detailed balance principle" [35], that means the equations of the density matrix must be of the form of: (1) Pauli master equations for the diagonal elements, and (2) Bloch-Feynman equations for the non-diagonal ones. In Sec. II we consider the Hamiltonian of the total system, in Sec. III we derive the quantum master equation, in Sec. IV we discuss the agreement with the detailed balance principle, in Sec. V we derive an evolution equation describing the environment dynamics correlated with the system dynamics, in Sec. VI we apply our quantum master equation to super radiance, in Sec. VII we derive the master equation of a harmonic oscillator and compare this equation with the generally accepted equation (8), and Sec. VIII is for conclusions.

2. DISSIPATIVE HAMILTONIAN AND STATES

We consider a system of Z charged Fermions of Hamiltonian H^S, with the coordinates $\vec{r}_n$ and momenta $\vec{p}_n$ $(n=1,2\ldots Z)$ in a dissipative environment of Hamiltonian H^E. The environment includes a system of other Fermions of Hamiltonian H^F, a system of Bosons of Hamiltonian H^B, and the free electromagnetic field of Hamiltonian H^{EM}, so that the Hamiltonian of the total system is:

$$H^T = H^S + V^E + H^E \equiv H^S + V^F + V^B + V^{EM} + H^F + H^B + H^{EM}, \qquad (12)$$

where $V^E = V^F + V^B + V^{EM}$ is the interaction Hamiltonian of the system of Hamiltonian H^S with its environment of Hamiltonian H^E. This approach does not necessarily mean that the environment is much larger than the system of interest, but only that the density of the environment states is much larger than the density of the system states, so that any single-particle transition $|j\rangle \to |i\rangle$ of the system is assisted with a certain probability by a single-particle transition $|\beta\rangle \to |\alpha\rangle$ of the environment satisfying the resonance condition $\omega_{\alpha\beta} = \omega_{ji}$, the total energy of this process being conserved. If the system and the environmental Fermions are placed in rather separated potential wells, the dissipative coupling is entirely described by the electric interaction, while the overlap of the wave-functions can be neglected. In this case, no anti-symmetrization of the system and environment states or transition between the systemand the environment states is taken into account, the number of the system particles being conserved. With the single-particle potential $U^{(1)}(\vec{r}_n)$ including the self-consistent field and the residual two-body potential $U^{(2)}(\vec{r}_n,\vec{r}_m)$, Eq. (12) takes a form

$$H^T = \sum_{n=1}^{Z} \frac{(\vec{p}_n - e\vec{A}^{EM})^2}{2M} + \sum_{n=1}^{Z} U^{(1)}(\vec{r}_n) + \frac{1}{2}\sum_{n,m=1}^{Z} U^{(2)}(\vec{r}_n, \vec{r}_m) + \\ + V^F + V^B + H^E, \tag{13}$$

having as terms the interaction potential with the free electromagnetic field

$$V^{EM} = -\frac{e}{M}\sum_{n=1}^{Z} \vec{p}_n \vec{A}^{EM}, \tag{14}$$

the dissipative potential with the environment of Fermions

$$V^F = \sum_{ij,\alpha\beta} \langle \alpha i | V^F | \beta j \rangle c_\alpha^+ c_i^+ c_j c_\beta \equiv \hbar \sum_{ij} \Gamma_{ij}^F c_i^+ c_j, \tag{15}$$

and a similar expression for the dissipative potential V^B with the environment of Bosons (another than the free electromagnetic field). In Eq. (14),

$$\vec{A}^{EM} = \sum_{\nu} \vec{A}_\nu, \quad \vec{A}_\nu = \vec{1}_\nu \sqrt{\frac{\hbar}{2\varepsilon V \omega_\nu}} (a_\nu + a_\nu^+) \tag{16}$$

is the vector potential of the free electromagnetic field bath with the components $\vec{A}_\nu$ of the field modes ν, where $a_\nu^+ - a_\nu$ are the creation-annihilation operators in the quantization volume $\mathcal{V}$.

To reduce (14) to an expression similar to (15), we introduce the momentum in the second quantization:

$$\vec{p} \equiv \sum_n \vec{p}_n = iM \sum_{ij} \omega_{ij} \vec{r}_{ij} c_i^+ c_j, \tag{17}$$

where

$$\vec{r}_{ij} = \langle i | \vec{r} | j \rangle = \int \psi_i^*(\vec{r}) \vec{r} \psi_j(\vec{r}) d^3\vec{r} \tag{18}$$

is the dipole moment. We obtain

$$V^{EM} = \hbar \sum_{ij} \Gamma_{ij}^{EM} c_i^+ c_j, \tag{19}$$

with field operators

$$\Gamma_{ij}^{EM} = \sum_{\nu} \Gamma_{ij}^{(\nu)}, \quad \Gamma_{ij}^{(\nu)} = \mp i \omega_{ij} G_{ij}^{(\nu)} (a_\nu + a_\nu^+), \tag{20}$$

depending on coupling coefficients

$$G_{ij}^{(\nu)} = \vec{r}_{ij} \vec{1}_\nu \sqrt{\frac{2\pi\alpha_0 c}{V\omega_\nu}}, \quad \alpha_0 = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}. \tag{21}$$

In (20), the minus/plus sign corresponds to the positive/negative charge of the Fermions. In this way, the dissipative potential takes a general form

$$V^E = V^F + V^B + V^{EM} = \hbar \sum_{ij} \Gamma_{ij} c_i^+ c_j, \tag{22}$$

depending on operators that describe a complex environment of Fermions, Bosons, and free electromagnetic field:

$$\Gamma_{ij} = \Gamma_{ij}^F + \Gamma_{ij}^B + \Gamma_{ij}^{EM}. \tag{23}$$

In the total Hamiltonian (13) we also distinguish the Hamiltonian of the system

$$H^S = \sum_{n=1}^{Z} \frac{\vec{p}_n^2}{2M} + \sum_{n=1}^{Z} U^{(1)}(\vec{r}_n) + \frac{1}{2} \sum_{n,m=1}^{Z} U^{(2)}(\vec{r}_n, \vec{r}_m), \tag{24}$$

and the Hamiltonian of the free electromagnetic field

$$H^{EM} = \sum_{\nu} H^{(\nu)}, \quad H^{(\nu)} = \hbar\omega_\nu (a_\nu^+ a_\nu + \frac{1}{2}). \tag{25}$$

According to the shell-model of a system of interacting Fermions, the Hamiltonian (24) is of a form

$$H^S = H_0^S + V^R, \tag{26}$$

with a single-particle diagonal term

$$H_0^S = \sum_i \varepsilon_i c_i^+ c_i, \tag{27}$$

and a residual two-body potential

$$V^R = \frac{1}{4} \sum_{ijkl} \langle ij | V^R | kl \rangle c_i^+ c_j^+ c_l c_k, \tag{28}$$

where the eigenstates $|i\rangle$ correspond to the one-particle potential $U^{(1)}(\vec{r}_n)$ for a self-consistent field of the other particles in the ground states, and the matrix elements $\langle ij | V^R | kl \rangle$ are anti-symmetric functions of $U^{(2)}(\vec{r}_n, \vec{r}_m)$.

3. QUANTUM MASTER EQUATION

As it has been demonstrated by Ford, Lewis and O'Connell [8, 29], the preservation of the quantum-mechanical conditions in the derivation of a master equation does not necessarily require the semi-group formalism, but it is also

possible in a physical derivation, by using an algorithm in three steps: (1) a series expansion of the total density matrix and of the equation of motion as functions of the dissipative coupling strength, (2) a tracing over the environment states, and (3) an averaging of the rapid oscillations induced by the system-environment interaction.

We consider the density operator $\chi(t)$ of the total system with the Hamiltonian (12), and the reduced density

$$\rho(t) = Tr_E\{\chi(t)\} \tag{29}$$

over the environment states. The total density $\chi(t)$ satisfies the equation of motion:

$$\frac{d\tilde{\chi}}{dt} = -\frac{i}{\hbar}[\varepsilon\tilde{V}^R(t) + \varepsilon\tilde{V}^E(t), \tilde{\chi}(t)], \tag{30}$$

where tilde denotes operators in the interaction picture of the system and environment,

$$\tilde{\chi}(t) = e^{\frac{i}{\hbar}(H^E + H_0^S)t}\chi(t)e^{-\frac{i}{\hbar}(H_0^S + H^E)t}, \tag{31}$$

while ε is a strength parameter used only to handle the orders of a series expansion of this density. Considering the dissipative environment in the initial state R, the total density of the system may be considered of the form [29]:

$$\tilde{\chi}(t) = R \otimes \tilde{\rho}(t) + \varepsilon\tilde{\chi}^{(1)}(t) + \varepsilon^2\tilde{\chi}^{(2)}(t) + \ldots, \tag{32}$$

where $\tilde{\chi}^{(1)}(t)$, $\tilde{\chi}^{(2)}(t)$ describe modifications of the environment during the evolution of the system. The first term of this expression corresponds to an approximation where the environment is described by a constant separable state R during the whole evolution of the system state $\tilde{\rho}(t)$. The higher-order terms, that satisfy the normalization relations

$$Tr_E\{\chi^{(1)}(t)\} = Tr_E\{\chi^{(2)}(t)\} = \ldots = 0, \tag{33}$$

represent non-separable state components, describing transitions of the environment correlated with the transitions of the system. We also consider a series expansion of the equation of motion [29],

$$\frac{d\tilde{\rho}}{dt} = \varepsilon\tilde{B}^{(1)}(\tilde{\rho}(t), t) + \varepsilon^2\tilde{B}^{(2)}(\tilde{\rho}(t), t) + \ldots, \tag{34}$$

and, from (30) with (32) and (34), we get a system of coupled equations. Calculating partial traces over the environment states with the normalization conditions (33), from these equations we successively obtain the terms of the equation of motion (34),

$$\tilde{B}^{(1)}(\tilde{\rho}(t),t) = -\frac{i}{\hbar} Tr_E[\tilde{V}^R(t) + \tilde{V}^E(t), R \otimes \tilde{\rho}(t)] \tag{35}$$

$$\tilde{B}^{(2)}(\tilde{\rho}(t),t) = -\frac{i}{\hbar} Tr_E[\tilde{V}^R(t) + \tilde{V}^E(t), \tilde{\chi}^{(1)}(t)] \tag{36}$$

$$\tilde{B}^{(3)}(\tilde{\rho}(t),t) = -\frac{i}{\hbar} Tr_E[\tilde{V}^R(t) + \tilde{V}^E(t), \tilde{\chi}^{(2)}(t)] \tag{37}$$

…,

while integrating with time, we get the excitation terms of the total density (32):

$$\tilde{\chi}^{(1)}(t) = \int_0^t \left\{ -\frac{i}{\hbar} [\tilde{V}^R(t') + \tilde{V}^E(t'), R \otimes \tilde{\rho}(t')] - R \otimes \tilde{B}^{(1)}(\tilde{\rho}(t'),t') \right\} dt' \tag{38}$$

$$\tilde{\chi}^{(2)}(t) = \int_0^t \left\{ -\frac{i}{\hbar} [\tilde{V}^R(t') + \tilde{V}^E(t'), \tilde{\chi}^{(1)}(t')] - R \otimes \tilde{B}^{(2)}(\tilde{\rho}(t'),t') \right\} dt' \tag{39}$$

… .

The first-order equation (35) represents the system evolution when the environment is considered in a stationary state R, while in the higher-order terms (36), (37), …, environment excitations according to (38), (39), … are taken into account.

For the environment we consider an equilibrium initial state

$$R = R^F \otimes R^B \otimes R^{EM}, \qquad R^{EM} = \prod_{\nu} R_\nu \tag{40}$$

in a diagonal representation, with the expression

$$R^F = \frac{1}{Y^F} \sum_{\alpha} \varepsilon_\alpha c_\alpha^+ c_\alpha \tag{41}$$

for an environment of Y^F Fermions, and a similar expression for R^B, while for the electromagnetic field modes ν we have the expression

$$R_\nu = \sum_{n_\nu} (1 - e^{-\hbar\omega_\nu/T}) e^{-n_\nu \hbar\omega_\nu/T} \left| n_\nu \right\rangle \left\langle n_\nu \right| \tag{42}$$

depending the number of photons n_ν and on temperature T. Taking into account that, with the non-diagonal operators Γ_{ij} and the diagonal operators R,

$$Tr_E[\Gamma_{ij} R] = 0, \tag{43}$$

from (35) with (22–23), (40), and (42), we obtain

$$Tr_E[\tilde{V}^E(t), R \otimes \tilde{\rho}(t)] = 0, \tag{44}$$

so that in the first-order term of the reduced density equation of motion only the Hamiltonian part remains:

$$\tilde{B}^{(1)}(\tilde{\rho}(t),t)=-\frac{i}{\hbar}[\tilde{V}^{R}(t),\tilde{\rho}(t)]. \tag{45}$$

With (45), the first-order excitation term (38) of the total density is

$$\tilde{\chi}^{(1)}(t)=-\frac{i}{\hbar}\int_{0}^{t}[\tilde{V}^{E}(t'),R\otimes\tilde{\rho}(t')]dt', \tag{46}$$

while, with the relation (44), the second-order term (36) of the reduced density equation takes a form

$$\begin{aligned}\tilde{B}^{(2)}(\tilde{\rho}(t),t)=\sum_{ijkl}\int_{0}^{t}\{&Tr_{E}[\tilde{\Gamma}_{ij}(t')R\tilde{\Gamma}_{kl}(t)][\tilde{c}_{i}^{+}(t')\tilde{c}_{j}(t')\tilde{\rho}(t'),\tilde{c}_{k}^{+}(t)\tilde{c}_{l}(t)]+\\&+Tr_{E}[\tilde{\Gamma}_{ij}(t)R\tilde{\Gamma}_{kl}(t')][\tilde{c}_{i}^{+}(t)\tilde{c}_{j}(t),\tilde{\rho}(t')\tilde{c}_{k}^{+}(t')\tilde{c}_{l}(t')]\}dt'\end{aligned} \tag{47}$$

with three terms of interaction with the environment components

$$B^{(2)}=B_{F}^{(2)}+B_{B}^{(2)}+B_{EM}^{(2)}, \tag{48}$$

corresponding to the three terms of the expression (23).

In these expressions we have in view that the transition operators of the system

$$\tilde{c}_{i}^{+}(t)\tilde{c}_{j}(t)=e^{i\omega_{ij}t}c_{i}^{+}c_{j} \tag{49}$$

and the transition operators of the environment

$$\tilde{c}_{\alpha}^{+}(t)\tilde{c}_{\beta}(t)=e^{i\omega_{\alpha\beta}t}c_{\alpha}^{+}c_{\beta},\qquad \tilde{a}_{\nu}(t)=e^{-i\omega_{\nu}t}a_{\nu} \tag{50}$$

are rapidly-varying in time, while $\tilde{\rho}(t')$ is a slowly-varying function. We also consider a time-symmetry leading only to terms that depend only on time intervals $t-t'$ in the time-integrals, the terms depending on the origin of time being swept out in the summations over the high density spectrum of the environment states. Thus, Eq. (47) mainly describes single-particle transitions $|j\rangle\leftrightarrow|i\rangle$ of the system, correlated with transitions $|\beta\rangle\leftrightarrow|\alpha\rangle$ or $|n_{\nu}\rangle\leftrightarrow|n_{\nu}+1\rangle$ of the environment,

$$\omega_{ji}\approx\omega_{\nu}\quad\text{or}\quad\omega_{ji}\approx\omega_{\alpha\beta}, \tag{51}$$

while the summations over the environment states can be transformed into integrals that depend on densities of states considered as environment characteristics.

For the free electromagnetic field component of the environment, the density of states is obtained by quantization in an arbitrary volume $\mathcal{V}$:

$$\sum_{\nu} = \frac{\mathcal{V}}{\pi^2 c^3} \int_0^{\infty} d\omega_{\nu} \cdot \omega_{\nu}^2. \tag{52}$$

Thus, in (47) we get integrals of rapidly-varying functions that can be evaluated as

$$\int_0^{\infty} d\omega_{\nu} \int_0^t dt' e^{i(\omega_{ji} - \omega_{\nu})(t-t')} = \pi. \tag{53}$$

and, by introducing (20–21), (40), (42) and (52) in (47), one gets:

$$\begin{aligned} &\tilde{B}_{EM}^{(2)}(\tilde{\rho}(t), t) = \frac{2\alpha_0}{\pi c^2} \sum_{ij} \omega_{ji}^2 \vec{r}_{ij}^2 \int_0^{\infty} \omega_{\nu} d\omega_{\nu} \int_0^t dt' \cdot \\ &\cdot \left\{ \left[\left(\langle n_{\nu} \rangle + 1 \right) e^{i(\omega_{ji} - \omega_{\nu})(t-t')} + \langle n_{\nu} \rangle e^{i(\omega_{ji} + \omega_{\nu})(t-t')} \right] \cdot [c_i^+ c_j \tilde{\rho}(t'), c_j^+ c_i] + H.C. \right\}, \end{aligned} \tag{54}$$

where

$$\langle n_{\nu} \rangle = \frac{1}{e^{\hbar\omega_{\nu}/T} - 1} \tag{55}$$

is the photon number mean-value of the mode ν at temperature *T*. Since $\omega_{\nu} > 0$, we consider separate summations for $i < j$ and $i > j$, neglect the rapidly terms in $e^{i(\omega_{ji} + \omega_{\nu})(t-t')}$ and respectively $e^{i(\omega_{ij} + \omega_{\nu})(t-t')}$, and evaluate the double integrals of the other contributing terms according to (53). Thus, we get the equation

$$\begin{aligned} \tilde{B}_{EM}^{(2)}(\tilde{\rho}(t), t) = &\frac{2\alpha_0}{c^2} \sum_{i<j} \omega_{ji}^3 \vec{r}_{ij}^2 \left\{ \left(\langle n_{\nu} \rangle + 1 \right) [c_i^+ c_j \tilde{\rho}(t), c_j^+ c_i] + H.C. \right\} + \\ &+ \frac{2\alpha_0}{c^2} \sum_{i>j} \omega_{ij}^3 \vec{r}_{ij}^2 \left\{ \langle n_{\nu} \rangle [c_i^+ c_j \tilde{\rho}(t), c_j^+ c_i] + H.C. \right\}, \end{aligned} \tag{56}$$

with the quasi-resonance condition (51). Calculating the coefficients of this equation with (55) and (51), we obtain

$$\tilde{B}_{EM}^{(2)}(\tilde{\rho}(t), t) = \sum_{ij} \gamma_{ij} ([c_i^+ c_j \tilde{\rho}(t), c_j^+ c_i] + [c_i^+ c_j, \tilde{\rho}(t) c_j^+ c_i]), \tag{57}$$

where

$$\gamma_{ij} = \frac{2\alpha_0}{c^2 \hbar^3} \vec{r}_{ij}^2 \varepsilon_{ji}^3 \left(1 + \frac{1}{e^{\varepsilon_{ji}/T} - 1} \right). \tag{58}$$

We note that the dissipative coefficients (58) remain positive when the transition energy change the sign. These coefficients correctly describe the dissipative

processes, *i.e.* γ_{ij} take significant values only for transitions $|j\rangle \to |i\rangle$ with $\varepsilon_{ji} > 0$, while for the transitions $\varepsilon_{ji} < 0$ ($|\varepsilon_{ji}| \gg T$), $\gamma_{ij} \to 0$.

By similar calculations, we obtain dissipative coefficients for the interaction with the environment of Y^F Fermions

$$\begin{aligned} \lambda_{ij}^F &= \frac{\pi}{\hbar Y^F} \int \left|\langle \alpha i | V^F | \beta j \rangle\right|^2 [1 - f_\alpha^F(\varepsilon_\alpha)] f_\beta^F(\varepsilon_\beta) g_\alpha^F(\varepsilon_\alpha) g_\beta^F(\varepsilon_\beta) d\varepsilon_\beta, \\ \varepsilon_{\alpha\beta} &= \varepsilon_{ji}, \end{aligned} \tag{59}$$

and with the environment of Y^B Bosons

$$\begin{aligned} \lambda_{ij}^B &= \frac{\pi}{\hbar Y^B} \int \left|\langle \alpha i | V^B | \beta j \rangle\right|^2 [1 + f_\alpha^B(\varepsilon_\alpha)] f_\beta^B(\varepsilon_\beta) g_\alpha^B(\varepsilon_\alpha) g_\beta^B(\varepsilon_\beta) d\varepsilon_\beta, \\ \varepsilon_{\alpha\beta} &= \varepsilon_{ji}. \end{aligned} \tag{60}$$

In these expressions, V^F, V^B are dissipative two-body potentials, $g_\alpha^F(\varepsilon_\alpha)$, $g_\beta^F(\varepsilon_\beta)$, $g_\alpha^B(\varepsilon_\alpha)$, $g_\beta^B(\varepsilon_\beta)$ are densities of the environment states, and $f_\alpha^F(\varepsilon_\alpha)$, $f_\beta^F(\varepsilon_\beta)$, $f_\alpha^B(\varepsilon_\alpha)$, $f_\beta^B(\varepsilon_\beta)$ are occupation probabilities of these states, normalized to the total numbers of the environment particles Y^F and respectively Y^B. Using (27), (28), (45), and (57–60), from (34) we obtain the second-order master equation

$$\begin{aligned} \frac{d}{dt}\rho(t) = &-\frac{i}{\hbar}\sum_i \varepsilon_i [c_i^+ c_i, \rho(t)] - \frac{i}{4\hbar}\sum_{ijkl} V_{ijkl}^R [c_i^+ c_j^+ c_l c_k, \rho(t)] + \\ &+ \sum_{ij} \lambda_{ij} ([c_i^+ c_j \rho(t), c_j^+ c_i] + [c_i^+ c_j, \rho(t) c_j^+ c_i]), \end{aligned} \tag{61}$$

with the dissipative coefficients

$$\lambda_{ij} = \lambda_{ij}^F + \lambda_{ij}^B + \gamma_{ij}. \tag{62}$$

In this equation we distinguish three terms: (1) a Hamiltonian term for the single-particle eigenstates of the system, (2) a Hamiltonian term for transitions among these states due to the residual potential V^R, and (3) a dissipative term, describing a spontaneous decay through interaction with the three components of the environment: (1) many other Fermions of the physical structure, (2) Bosons, as the physical structure vibrations, (3) the free electromagnetic field. A transition $|j\rangle \to |i\rangle$ of a system particle is correlated with a transition $|\beta\rangle \to |\alpha\rangle$ of an environment particle, $\omega_{ji} = \omega_{\alpha\beta}$ (resonant dissipation).

4. DISSIPATION COEFFICIENTS AND DYNAMICAL DETAILED BALANCE

In the following we neglect the residual potential V^R, that has been introduced here only to re-obtain this very important term of the shell model in the derivation of the quantum master equation (61). We obtain equations of matrix elements

$$\frac{d}{dt}\rho_{mn}(t) = -i\omega_{mn}\rho_{mn}(t) + \sum_j [2\delta_{mn}\lambda_{mj}\rho_{jj}(t) - (\lambda_{jm} + \lambda_{jn})\rho_{mn}(t)] \tag{63}$$

that, for the diagonal ones, lead to Pauli master equations

$$\frac{d}{dt}\rho_{nn}(t) = 2\sum_j [\lambda_{nj}\rho_{jj}(t) - \lambda_{jn}\rho_{nn}(t)], \tag{64}$$

while, for the non-diagonal matrix elements, take the form of the damped Bloch-Feynman equations [36]

$$\frac{d}{dt}\rho_{mn}(t) = -i\omega_{mn}\rho_{mn}(t) - \Lambda_{mn}\rho_{mn}(t) \tag{65}$$

with dephasing rates

$$\Lambda_{mn} = \sum_j (\lambda_{jm} + \lambda_{jn}). \tag{66}$$

We notice that Eqs. (65) do not contain any dissipative couplings between different transition elements as $\rho_{mn}(t)$ with $\rho_{m-1,n-1}(t)$ or $\rho_{m+1,n+1}(t)$), such as the master equations (4) and (8) do. Such couplings are not revealed in the most studies of quantum optics [36].

Considering the initial distribution $f_\beta^F(\varepsilon_\beta)$ of the environment states as rapidly-decreasing in the proximity of the Fermi level, $\varepsilon_\beta \approx 0$, and the final distributions $f_\alpha^F(\varepsilon_\alpha)$, $g_\alpha^F(\varepsilon_\alpha)$ as slowly-varying functions of energy $\varepsilon_j - \varepsilon_i = \varepsilon_\alpha - \varepsilon_\beta \approx \varepsilon_\alpha$, from the coefficients (59) and (60) for the direct and reverse transitions, one obtains simpler expressions of these coefficients. For an environment of Fermions

$$\lambda_{ij} = \lambda_{ij}^F = \frac{\pi}{\hbar}\left|\langle \alpha i | V^F | \beta j \rangle\right|^2 [1 - f_\alpha^F(\varepsilon_{ji})] g_\alpha^F(\varepsilon_{ji}) \tag{67a}$$

$$\lambda_{ji} = \lambda_{ji}^F = \frac{\pi}{\hbar}\left|\langle \alpha i | V^F | \beta j \rangle\right|^2 f_\alpha^F(\varepsilon_{ji}) g_\alpha^F(\varepsilon_{ji}), \tag{67b}$$

and for an environment of Bosons

$$\lambda_{ij} = \lambda_{ij}^B = \frac{\pi}{\hbar}\left|\langle \alpha i | V^B | \beta j \rangle\right|^2 [1 + f_\alpha^B(\varepsilon_{ji})] g_\alpha^B(\varepsilon_{ji}) \tag{68a}$$

$$\lambda_{ji} = \lambda_{ji}^B = \frac{\pi}{\hbar}\left|\langle \alpha i | V^B | \beta j \rangle\right|^2 f_\alpha^B(\varepsilon_{ji}) g_\alpha^B(\varepsilon_{ji}). \tag{68b}$$

According to Eqs. (64) with the coefficients (67), (68) and (58) the diagonal matrix elements of the density satisfy the conditions of a detailed balance [7–9] for an environment of Fermions

$$\frac{\rho_{jj}(\infty)}{\rho_{nn}(\infty)} = \frac{\lambda_{jn}}{\lambda_{nj}} = \frac{f_\alpha^F(\varepsilon_{jn})}{1 - f_\alpha^F(\varepsilon_{jn})} = e^{-\varepsilon_{jn}/T}, \tag{69}$$

and Bosons, or, particularly, electromagnetic field

$$\frac{\rho_{jj}(\infty)}{\rho_{nn}(\infty)} = \frac{\lambda_{jn}}{\lambda_{nj}} = \frac{f_\alpha^B(\varepsilon_{jn})}{1 + f_\alpha^B(\varepsilon_{jn})} = e^{-\varepsilon_{jn}/T}, \tag{70}$$

while the non-diagonal matrix elements are separately damped according to the generally-accepted Bloch-Feynman equations (dynamical detailed balance).

5. THE ENVIRONMENT DYNAMICS

The quantum master equation (61) describes the evolution of a system of Fermions under the action of the dissipative environment. For a complete description, we have also to calculate the environment evolution under the action of the system. In the second-order approximation, from (32) we obtain the environment density matrix

$$\tilde{\rho}^E(t) \equiv Tr_S\{\tilde{\chi}(t)\} = R + Tr_S\left\{\tilde{\chi}^{(1)}(t)\right\} + Tr_S\left\{\tilde{\chi}^{(2)}(t)\right\} + \ldots \tag{71}$$

as a solution of the equations (35), (36) and (38). From these equations we get:

$$\frac{d}{dt} Tr_S\left\{\tilde{\chi}^{(1)}(t)\right\} = -i\sum_{ij} Tr_S\left[\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t), R\tilde{\rho}(t)\right] \tag{72a}$$

$$\begin{aligned}\frac{d}{dt} Tr_S\left\{\tilde{\chi}^{(2)}(t)\right\} = \sum_{ijkl}\int_0^t Tr_S\left\{\left[\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t), R\zeta_{kl}\left[\tilde{c}_k^+(t')\tilde{c}_l(t'), \tilde{\rho}(t')\right]\right] - \right. \\ \left. -\left[\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t), \left[\tilde{\Gamma}_{kl}(t')\tilde{c}_k^+(t')\tilde{c}_l(t'), R\tilde{\rho}(t')\right]\right]\right\} dt'.\end{aligned} \tag{72b}$$

Here we consider only the interaction picture, since for the environment we are interested only in the slowly-varying part of the time evolution. Besides the

equilibrium component *R*, the environment density matrix $\tilde{\rho}^E(t)$ includes three terms depending on the system density matrix $\tilde{\rho}(t)$: (1) a term of the first-order in the system-environment correlations $\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t)$, (2) a term of the second-order in the system-environment correlations andin the self-consistent field matrix elements, and (3) a term of the second-order in the system-environment correlations. We find that only the terms exclusively depending on the system-environment correlations, leading to Markovian terms, remain in the final result, while the terms depending on the self-consistent field reduce to zero.

With the explicit expressions of the time-dependent operators in the interaction picture, Eqs. (72) become:

$$\frac{d}{dt}Tr_S\left\{\tilde{\chi}^{(1)}(t)\right\} = -\frac{i}{\hbar}\sum_{ij}\sum_{\alpha\beta}\langle\alpha i|V^F|\beta j\rangle e^{i(\omega_{\alpha\beta}-\omega_{ji})t}[c_\alpha^+c_\beta, R]\tilde{\rho}_{ji}(t) \tag{73a}$$

$$\begin{aligned}\frac{d}{dt}Tr_S\left\{\tilde{\chi}^{(2)}(t)\right\} = -\frac{1}{\hbar^2}\sum_{ij}\sum_{\alpha\beta}\int_0^t\left|\langle\alpha i|V^F|\beta j\rangle\right|^2 e^{i(\omega_{\alpha\beta}-\omega_{ji})(t-t')}\cdot\\ \cdot\left\{[c_\alpha^+c_\beta, c_\beta^+c_\alpha R]\tilde{\rho}_{ii}(t) - [c_\alpha^+c_\beta, Rc_\beta^+c_\alpha]\tilde{\rho}_{jj}(t)\right\}dt'\end{aligned} \tag{73b}$$

With the resonant condition and using Eq. (41) with the condition of a quasi-continuum spectrum of environment states, we obtain the environment master equation:

$$\begin{aligned}\frac{d}{dt}\tilde{\rho}^E(t) = -\frac{i}{\hbar Y^F}\sum_{ij}\tilde{\rho}_{ji}(t)\int_{(\beta)}\langle\alpha i|V^F|\beta j\rangle g_\beta(\varepsilon_\beta)c_\alpha^+c_\beta\cdot\\ \cdot\left\{\left[1-f_\alpha(\varepsilon_\alpha)\right]f_\beta(\varepsilon_\beta) - \left[1-f_\beta(\varepsilon_\beta)\right]f_\alpha(\varepsilon_\alpha)\right\}\Big|_{\omega_{\alpha\beta}=\omega_{ji}}d\varepsilon_\beta +\\ +\frac{\pi}{\hbar Y^F}\sum_{ij}\int_{(\beta)}\left|\langle\alpha i|V^F|\beta j\rangle\right|^2 g_\alpha(\varepsilon_\alpha)g_\beta(\varepsilon_\beta)\left(c_\alpha^+c_\alpha - c_\beta^+c_\beta\right)\cdot\\ \cdot\left\{\left[1-f_\alpha(\varepsilon_\alpha)\right]f_\beta(\varepsilon_\beta)\tilde{\rho}_{jj}(t) - \left[1-f_\beta(\varepsilon_\beta)\right]f_\alpha(\varepsilon_\alpha)\tilde{\rho}_{ii}(t)\right\}\Big|_{\omega_{\alpha\beta}=\omega_{ji}}d\varepsilon_\beta\end{aligned} \tag{74}$$

This equation satisfies basic conditions and is very transparent to physical interpretations. Really, the first term describes transitions $c_\alpha^+c_\beta$ of the environment correlated with the system transitions of probabilities $\tilde{\rho}_{ji}(t)$. The transition probability from an initial states $|\beta\rangle$ to a final states $|\alpha\rangle$ is considered as the difference between the direct and the reverse transitions between these states, that depend on the occupation probabilities $f_\alpha a(\varepsilon_\alpha)$, $f_\beta(\varepsilon_\beta)$ and the probabilities $1-f_\alpha(\varepsilon_\alpha)$, $1-f_\beta(\varepsilon_\beta)$ that these states are free. Since $\tilde{\rho}_{ji}(\infty)=0$, the asymptotic solution for $t\longrightarrow\infty$ of this term is also zero. The second term

describes population variations of the environment states while the system populations take non-equilibrium values. While the system populations tend to the equilibrium values

$$\frac{\tilde{\rho}_{jj}(\infty)}{\tilde{\rho}_{ii}(\infty)} = \frac{\left[1 - f_\beta(\varepsilon_\beta)\right] f_\alpha(\varepsilon_\alpha)}{\left[1 - f_\alpha(\varepsilon_\alpha)\right] f_\beta(\varepsilon_\beta)} = e^{-(\varepsilon_\alpha - \varepsilon_\beta)/T}, \tag{75}$$

this term tends also to zero. That means that while the system tends to equilibrium according to the detailed balance principle, the environment state tends also to equilibrium. We also notice that the population variations of two states $|\alpha\rangle$ and $|\beta\rangle$ are opposite to one another, the total population being conserved. While the system is in an excited state, $\tilde{\rho}_{jj}(t) > \tilde{\rho}_{ii}(t)$, the higher states $|\alpha\rangle$ are excited, while the lower states $|\beta\rangle$ are depleted.

6. SUPER RESONANCE OF A P-I-N STRUCTURE WITH QUANTUM DOTS

Despite a long history, the atom-field interaction is still an active field of investigation especially due to the dissipative processes [40], standing at the basis of important technical applications – an example is a new amplifying device basedon the dissipative super-radiant tunneling, recently proposed by M. Asada [41]. The super radiance of a system of electrons predicted by Dike [42] has been intensively studied taking into account various physical effects as: (1) the statistical distribution of the electron states [43], (2) level degeneracy effects [44, 45], (3) Langevin forces acting on the atomic system [46], (4) transverse effects [47], (5) competing of three-photon and one-photon transitions [48], (6) the super radiance spectrum [49], (7) existence of photon gaps [50–53], (8) spontaneously generated coherence effects [54], (9) super radiance suppression by scattering [55]. However, a detailed study of the super-radiant power as a function of the main physical characteristics of a specific system do not yet exist in literature, and the accordance of the dissipative super-radiant dynamics with the detailed balance principle [7, 8, 56] has not yet been discussed.

We investigate the super-radiance of a p-i-n semiconductor structure with quantum dots and a perfectly tuned micro-cavity (Fig. 1). Here the electrons decaying between well-determined energy levels on one side of this cavity build-up a super-radiant field transmitted out on the other side. The insulating region i enables a thorough control of the transition dipole $r_{01} \equiv \left|\langle 0|\vec{r}\,|1\rangle\right|$ that determines the matter-field coupling. We describe this system by a Hamiltonian

$$H = H_0^S + H^F + V, \tag{76}$$

with three terms: $H^S = \sum_i \varepsilon_i c_i^+ c_i$, $i = 0, 1$ for the system of electrons, $H^F = \hbar\omega\left(a^+ a + \frac{1}{2}\right)$ for the super-radiant field mode of frequency $\omega = \omega_{10}$, and the interaction potential

$$V = ie\sum_{ij} \omega_{ij} r_{ij} c_i^+ c_j A. \tag{77}$$

depending on the transition frequencies ω_{ij}, on the dipole moments r_{ij}, and on the potential vector

$$A = \frac{\hbar}{e} K\left(ae^{i\vec{k}\vec{r}} + a^+ e^{-i\vec{k}\vec{r}}\right), \tag{78}$$

where $K = \sqrt{\alpha_0 \frac{\lambda}{V}}$, while $\alpha_0 = \frac{e^2}{4\pi\varepsilon\hbar c}$, λ is the wavelength, and $\mathcal{V}$ is the quantization volume. We also take into account the existence of a complex dissipative coupling of the system described by the Hamiltonian H with (1) other electrons in the profound clusters of the n-region, (2) the crystalline lattice, (3) the free electromagnetic field coupled with the system of electrons, and (4) other dissipative elements of the semiconductor structure coupled with the super-radiant mode, that essentially determines the radiation process. The description of this process depends on the system-environment interaction model and on the approximations used to reduce a quantum dynamical equation to a master equation [57–60].

We consider the explicit quantum master equation 61 with the Hamiltonian (76):

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \sum_{i,j=1}^{N} \lambda_{ij}([c_i^+ c_j \rho(t), c_j^+ c_i] + [c_i^+ c_j, \rho(t) c_j^+ c_i]). \tag{79}$$

This equation for an N-level system has $N^2 - 1$ explicit dissipative coefficients (62).

Considering a two-level system, $i, j = 0, 1$ with negligible dimensions in comparison with the field wave length, and a perfectly tuned cavity, from (79), one obtains the Maxwell-Bloch equations

$$\frac{d}{dt}u(t) = -\gamma_\perp u(t) - gG(0,t)w(t) \tag{80a}$$

$$\frac{d}{dt}w(t) = -\gamma_\parallel [w(t) - w_0] + 2\Phi + (2 - T)gG(0,t)u(t) \tag{80b}$$

$$\frac{\partial}{\partial t}G(z,t)\Big|_{z=0} - \sqrt{T}c\frac{\partial}{\partial z}G(z,t)\Big|_{z=0} = -\gamma_{EM}G(z,t)\Big|_{z=0} - g\frac{\hbar\omega}{4\varepsilon\mathcal{V}}u(t) \tag{80c}$$

$$\frac{\partial}{\partial t}G(z,t)+c\frac{\partial}{\partial z}G(z,t)=0,\quad z>0 \tag{80d}$$

with the polarization $u(t)$, the population $w(t)$, the electromagnetic field amplitude $G(z,t)$, a particle flux Φ that could be injected in the device, the matter-field coupling coefficient $g=\frac{e}{\hbar}r_{01}$, the decay rate of the super-radiant mode γ_{EM}, and the dissipative coefficients

$$\gamma_{\perp}=\lambda_{01}+\lambda_{10}+\lambda_{00}+\lambda_{11} \tag{81a}$$

$$\gamma_{\parallel}=2(\lambda_{01}+\lambda_{10}) \tag{81b}$$

$$w_0=-\frac{1-e^{-\hbar\omega_0/T}}{1+e^{-\hbar\omega_0/T}}, \tag{81c}$$

where $\mathcal{T}$ is temperature. The second coefficient of the last term of Eq. (80b) takes into account the coupling with the two counter propagating waves in the cavity, while T describes the decrease of the matter-field coupling due to the field radiation through the output mirror. From Eq. (80c) we notice that the polarization $u(t)$ in the quantization volume $\mathcal{V}=1/N_e^{3/2}$, N_e being the number of quantum dots on the area unit, is a source for two field variations: (1) a time-variation of the field in the cavity $G(z,t)\big|_{z=0}$, and (2) a field flow through the mirror with the transmission coefficient T, where the transmitted field $G_T(z,t)=\sqrt{T}G(z,t)$ propagates according to Eq. (80d). For $T=0$ Eqs. (80) describe a closed cavity, while for $T=1$ they correspond to an open super radiant structure. When the dissipative coefficients are neglected, these equations satisfy conservation relations:

$$(2-T)V\frac{d}{dt}W(0,t)+\hbar\omega\frac{d}{dt}\rho_{11}(t)=0 \qquad \text{(energy)} \tag{82a}$$

$$(2-T)u^2(t)+w^2(t)=1 \qquad \text{(Bloch vector)}, \tag{82b}$$

where $W(0,t)=\varepsilon G^2(0,t)$ is the energy density of the electromagnetic field, while $\rho_{11}(t)=\frac{1}{2}[1+w(t)]$ is the population of the upper level. For $T=0$, these relations describe a closed cavity, with a factor 2 for the two counter propagating waves, while for $T=1$ they correspond to an open structure, with a single radiation mode. In comparison with other Maxwell-Bloch equations used in the super-radiance domain as Eqs. (47–49) in [46], our equations (80) have explicit expressions of the dissipative coefficients (81a), (81b) with (62), (67), (68), (58), that satisfy the detailed balance conditions (69), (70). More than that,

in (80c) we consider a current injection $I = eN_e\Phi$, and a dissipation of the super-radiant field, that, as it will be shown in the following, is necessary for the solution convergence. In the following, we take a resonance energy $\hbar\omega_{10} = 0.1$ eV, and calculate the power density (Poynting vector amplitude) $S(z,t) = TS_0(z,t)$ with $S_0(z,t) = cW(z,t) = c\varepsilon G^2(z,t)$ from Eqs. (80) with coefficients containing the main physical parameters of the system. From (81b) with (62)–(58), the decay rate gets a temperature-independent term and a temperature-dependent one, coming from the coupling of the system to the Fermion and respectively to the Boson part of the environment:

$$\gamma_{||} = \gamma_{||}^F + \gamma_{||}^B \cdot \frac{e^{\hbar\omega_0/\mathcal{T}} + 1}{e^{\hbar\omega_0/\mathcal{T}} - 1}, \tag{83}$$

that, essentially, means a temperature dependence given by the detailed balance relations (9), (10). For simplicity, we take into account only the Boson component of the decay rate that depends on temperature, and consider $\gamma_\perp = \gamma_{||}/2 = \gamma_{EM}$.

In Fig. 2 we represent the time evolution of an open structure ($T = 1$) for two cases: (a) under-damped, and (b) over-damped, while in Fig. 1 (b) the power density is represented in space, for the under-damped case with the same parameters but different values of the transmission coefficient *T*. In these representations, we considered the initial condition of a thermal state $w(0) = 0.4$, $\mathcal{T} = 273.15$ K, $\Phi = 0$, and an initial polarization $|u(0)| = \sqrt{w_0^2 - w^2(0)}$ in accordance with the equilibrium condition: $w(0) \to w_0 \Rightarrow u(0) \to 0$.

Fig. 3 illustrates other two effects decreasing the super-radiation field amplitude: (1) the propagation of this field in an open structure, that diminishes the matter-field coupling, and (2) the field mode dissipation. From Fig. 3 (a), we notice that for the tunneling of a packet of N_e electrons, the field propagation and dissipation have somehow similar effects, diminishing the pulse amplitude without changing its shape (green and magenta curves in comparison with the blue curve). However, from Fig. 3 (b) we notice that when a current *I* is injected in the structure, the two terms of the field equation, of propagation and of dissipation, have qualitatively different effects. Only through the field dissipation term (the magenta curve – $T = 0$, or the red curve – $T = 1$), Eqs. (80) have a finite asymptotic solution of the electromagnetic field density of energy

$$W(\infty) \equiv \varepsilon G^2(0,\infty) = \frac{1}{2-T} \cdot \left[\frac{\hbar\omega}{4\gamma_{EM}} \left(2\frac{I}{e} N_e^{1/2} + w_0 \gamma_{||} N_e^{3/2} \right) - \varepsilon \frac{\gamma_\perp \gamma_{||}}{g^2} \right], \tag{84}$$

while the population is

$$w(\infty)=\frac{w_0+\dfrac{2I}{eN_e\gamma_{||}}}{1+(2-T)\dfrac{g^2}{\gamma_\perp\gamma_{||}}G^2(0,\infty)}. \tag{85}$$

Otherwise, the continuous polarization created by the injected current I determines a continuous increase of the field (the blue curve – $T=0$, and the green curve – $T=1$), that is a non-physical solution – this increase is limited by field dissipation.Without a field dissipation ($\gamma_{EM}=0$), for an injected current above a threshold, the density of energy (84) becomes ∞, while the population difference (85) becomes 0, that means a violation of the detailed balance principle, that requires a population inversion for canceling the difference between the environment-assisted decays and excitations.

The basic problem discussed from the beginning of the super-radiance domain [42], and that continues to be of interest [53], is the exponential dependence of the super-radiant pulse amplitude on the number of tunneling electrons (no injected current – $I=0$). In Fig. 4 we represent this dependence for different decay rates. For a strongly dissipative case, the super-radiant exponent is approximately 3, while for a weekly dissipative case, this exponent decreases due to the Rabi oscillation that tends to limit the amplitude of the super-radiant pulse when the frequency of this oscillation increases with the quantum dot density.

7. HARMONIC OSCILLATOR IN BLACKBODY RADIATION FIELD

A harmonic oscillator has non zero dipole momenta only between successive levels, depending on the distance ω_0 between these levels:

$$r_{n+1,n}=r_{n,n+1}=\sqrt{\frac{n+1}{2}\cdot\frac{\hbar}{M\omega_0}}. \tag{86}$$

Using these expressions, the quantum master equation (61) with coefficients (58) can be particularized for a harmonic oscillator in a blackbody radiation field

$$\begin{aligned}\frac{d}{dt}\rho(t)=&-\frac{i}{\hbar}[H,\rho(t)]+\\&+\alpha_0\frac{\hbar\omega_0^2}{Mc^2}\sum_n\left\{\frac{n+1}{e^{\hbar\omega_0/T}-1}([c_{n+1}^+c_n\rho(t),c_n^+c_{n+1}]+[c_{n+1}^+c_n,\rho(t)c_n^+c_{n+1}])+\right.\\&\left.+n\left(1+\frac{1}{e^{\hbar\omega_0/T}-1}\right)([c_{n-1}^+c_n\rho(t),c_n^+c_{n-1}]+[c_{n-1}^+c_n,\rho(t)c_n^+c_{n-1}])\right\}.\end{aligned} \tag{87}$$

In comparison with Eq. (8), where the independent phenomenological parameters λ and T describe a constant damping of the harmonic oscillator, the new equation (87) describes transitions between successive levels with specific decay rates, that parametrically depend only on temperature. On this basis, damped Bloch equations that do not include any parasitic couplings of the non-diagonal matrix elements are obtained. In this formalism, couplings of the transitions between the equidistant levels of a harmonic oscillator [30] could be taken into account only as higher-order terms in the weak-coupling expansion of the dissipative dynamics [29].

8. CONCLUSIONS

We have obtained a quantum master equation with transition operators $c_i^+ c_j$, and microscopic coefficients depending on matrix elements, densities of the environment states, and occupation probabilities of these states. This equation describes the dynamics of an N-level system of Fermions in agreement with the principle of a dynamical detailed balance: (1) Pauli master equations for populations, (2) damped Bloch-Feynman equations for polarizations. If the initial energy of an environmental state is negligible in comparison with energy levels of the system, (3) a Boltzmann distribution of populations is obtained as an asymptotic solution – the detailed balance principle. In other words, this principle refers to approximate expressions of the excitation/decay rates that, neglecting the distribution of the environmental particles in the vicinity of the Fermi level, is valid only for low temperatures. Here we have obtained a more accurate description of the dissipative dynamics, that includes the high-temperature case by taking into account the whole spectral distribution of the environmental particles. Since the equilibrium populations, as asymptotic solution of the quantum master equation, are generated by the dissipative term of this equation, the detailed balance principle means in fact that dissipation is a necessary part of the quantum dynamics. In some application fields, as the matter-field interaction, the neglect of dissipation alter not only the accuracy of the results, but also the internal consistency of the physical description.

We discussed this equation in the context of other master equations describing the time-evolution of a system in accordance with the quantum-mechanical principles. For a harmonic oscillator coupled with the electromagnetic field, we obtained a master equation with a decay rate proportional to the excitation level, not with a constant one as is that of the well-known master equation with coordinate x and momentum p – the particles moving in the higher excitation states, with larger dipole momenta, have stronger couplings with the environment. Although the old equation in x and p satisfies the detailed balance conditions (1) and (3) for the diagonal matrix elements, it fails for the non-diagonal matrix

elements, including non-physical couplings between these elements. These couplings appear as an approximation effect, due to the utilization of only two operators x and p for the N^2-1 operators of an N-level system. The operators x and p, simultaneously including all the transition operators, do not enable the separation of the most probable resonant particle-particle couplings.

We derived a complementary master equation, describing the time evolution of the environment correlated with the system time evolution, and showed that tis equation also satisfies the detailed balance principle.

We derived Maxwell-Bloch equations for a super-radiant structure with quantum dots and a perfectly tuned cavity. In comparison with other Maxwell-Bloch equations previously used in the super-radiance domain, these equations have explicit microscopic coefficients, with a temperature dependence according to the detailed balance conditions, and include an injected current and a dissipation of the field that is necessary for their internal consistency. We investigated the dissipative super-radiant tunneling having in view principal characteristics that determine the super-radiant power. For realistic values of the system parameters we obtained super-radiant power densities comparable to the power density provided by Sun at the level of our planet, that is approximately 2 kW/m^2, thus suggesting the application to an efficient transformation of the solar power to micro-waves, while this efficiency is monitored in the framework of a microscopic model.

In this approach, we obtained a master equation in the second-order approximation that, describing single-particle transitions, is valid only for a weak dissipative coupling. A stronger dissipative coupling can be taken into account in a higher-order approximation, describing correlated transitions of Fermions.

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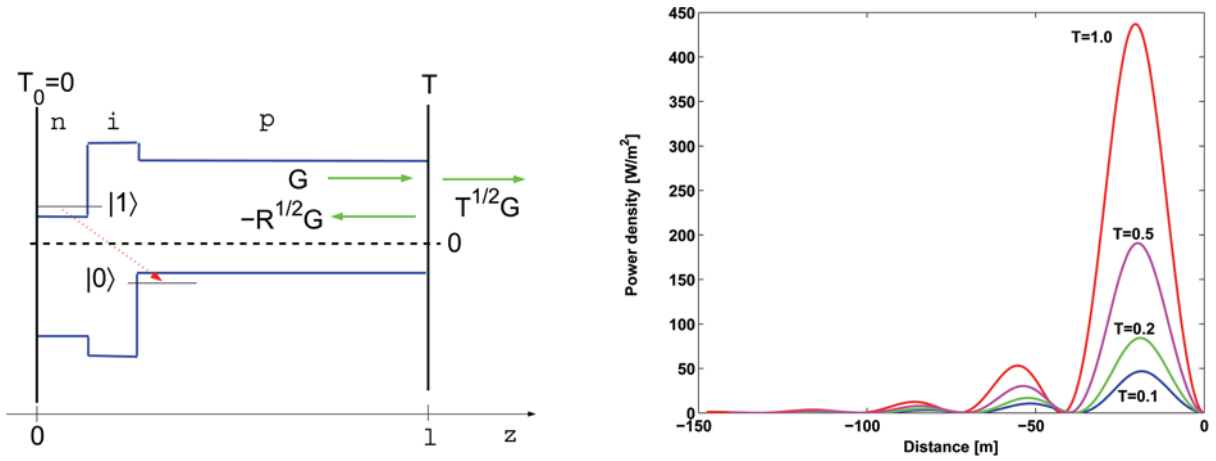
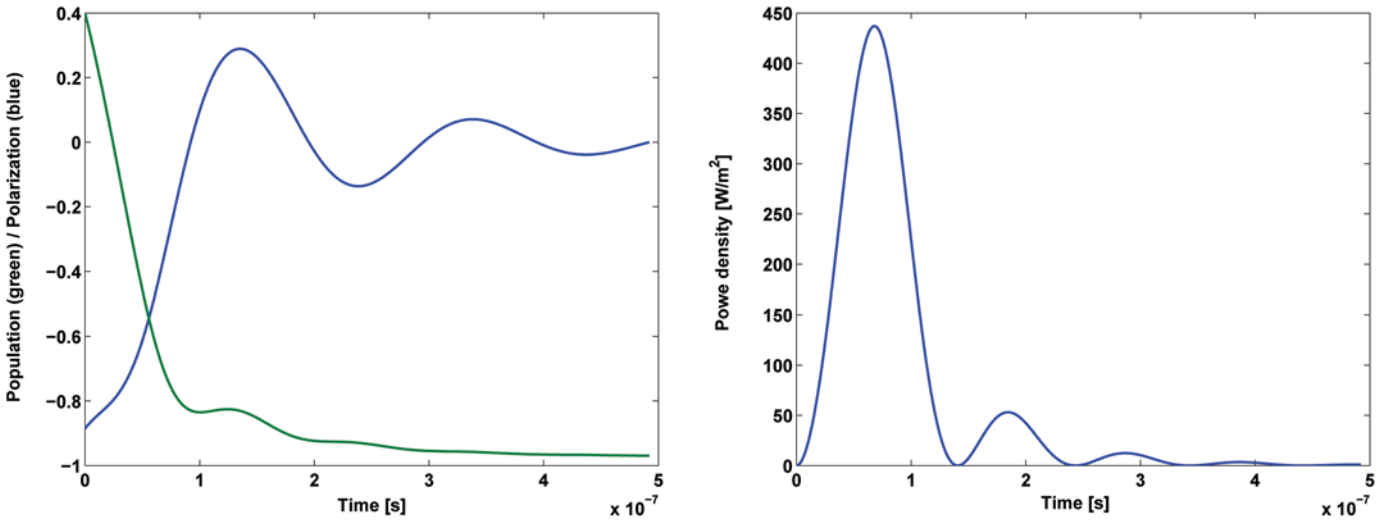


(a) Super-radiant semiconductor structure.

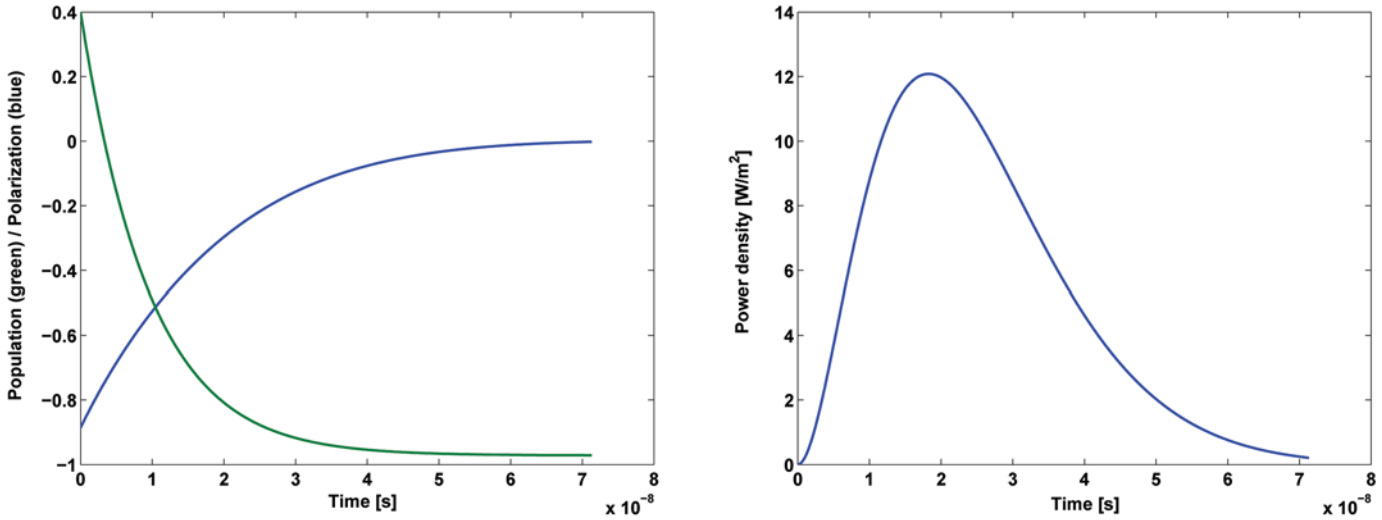
(b) Super-radiant electromagnetic pulse.

Fig. 1 – Super-radiant decay in a quantum dot p-i-n structure micro cavity.



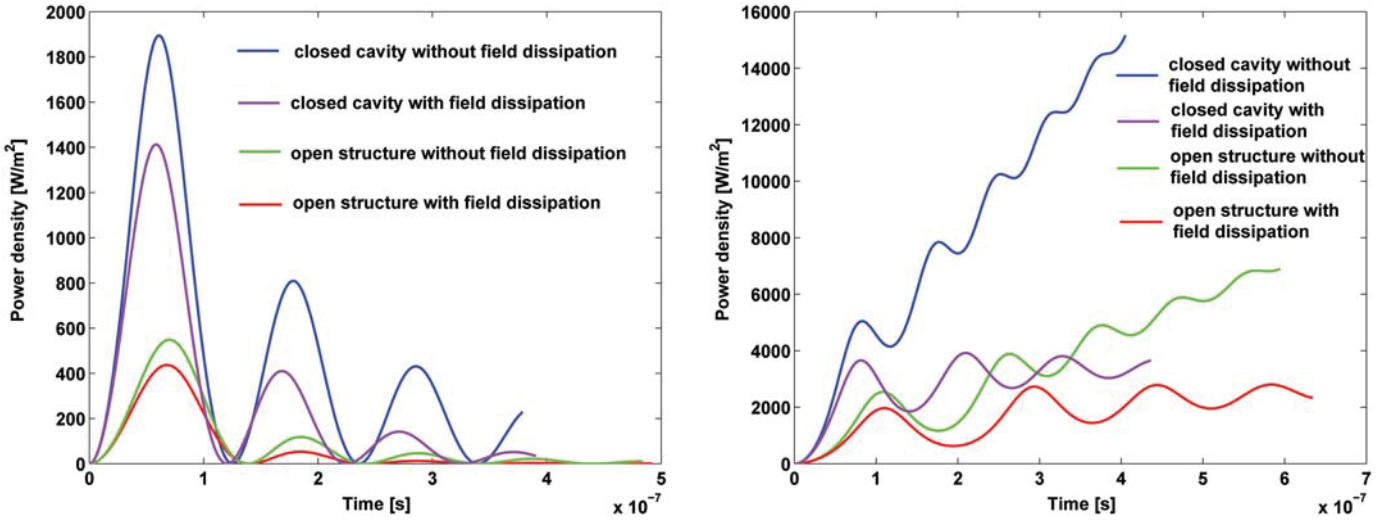
a) $\gamma_{\|}^{B} = 10^{7}\ \mathrm{s}^{-1}$, $\gamma_{\|}^{F} = 0$.

Fig. 2a



b) $\gamma_{\|}^{B} = 10^{8}\ \mathrm{s}^{-1},\ \gamma_{\|}^{F} = 0.$

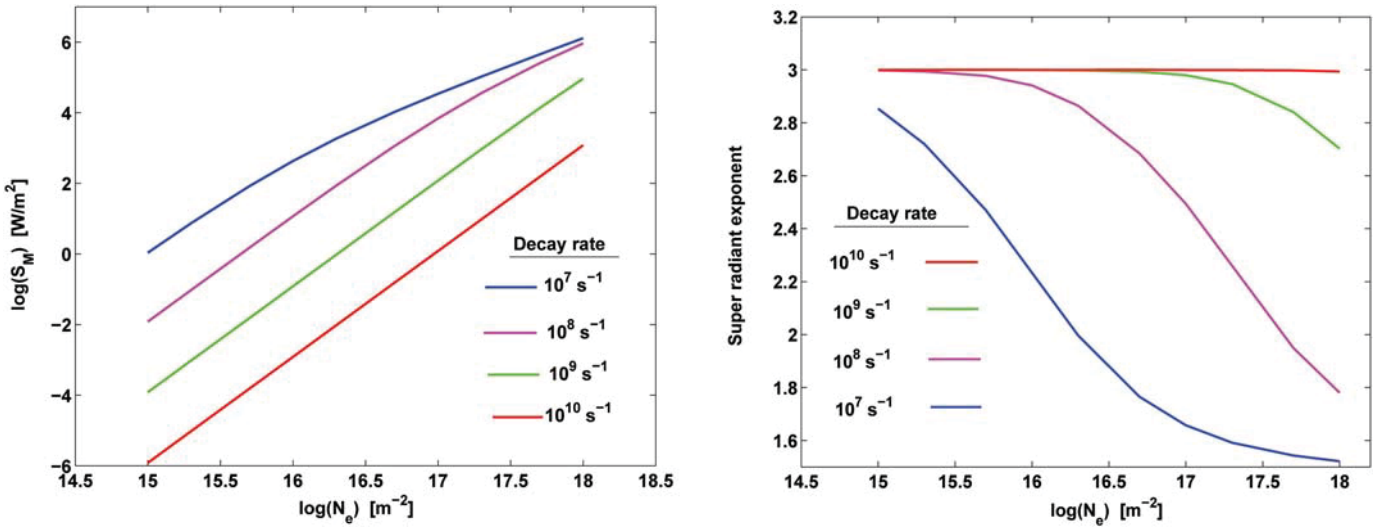
Fig. 2 – The population/polarization decay generating a super-radiant pulse in an open structure ($T = 1$), for an initial population $w(0) = 0.4$, a number of electrons (density of quantum dots) $N_e = 10^{16}$ electrons/m^2, a dipole moment $r_{01} = 10^{-6}$ nm and two values of the decay rate $\gamma_{\|}^{B}$ while $\mathcal{T} = 273.15$ K.



(a) Quantum tunneling without injected current ($\Phi = 0$).

(b) Quantum tunneling with an injected current $I = eN_e\Phi = 20$ mA/mm^2.

Fig. 3 – Super radiant pulse generation in quantum tunneling with and without injected current, for $w(0) = 0.4$, $N_e = 10^{16}$ quantum dots/m^2, $r_{01} = 10^{-6}$ nm, and $\gamma_{\|}^{B} = 10^{7}$ s^{-1}, $\gamma_{\|}^{F} = 0$.



(a) Absolute values.

(b) Super-radiant exponent.

Fig. 4 – The N_e-dependence of the super-radiant pulse amplitude S_M for different values of decay rate for a dipole moment $r_{01} = 10^6$ nm, $N_e = 10^{16}$ quantum dots/m^2, and $w(0) = 0.4$.