

CYCLIC UNIVERSES FROM TORUS GEOMETRY

R. MURDZEK

Physics Department, "Al. I. Cuza" University
rmurdzek@yahoo.com*Received April 27, 2006*

The models of cyclic universes emerge in a natural way from a doughnut-like space-time. The model is at least interesting because it allows us to approach most of fundamental cosmological problems from the perspective of the string theory.

Key words: Cosmology-general; M-theory; Cosmology-models.

1. INTRODUCTION

In the last twenty years, the Big-Bang and the inflationary model have made predictions which have been successfully confirmed by the observations. But they also come with some unsolved problems such as the horizon, flatness or the origin of structure problem.

Most recently, the observations seem to suggest that the universe rate of expansion is growing. At the first look, is just like there is a negative pressure which acts like an anti-gravitational field on large scales. Named "dark energy", it is effectively added into theory but anybody who knows what it really means.

On the other hand, the string theory appears to reborn and its modern form, the brane theory, allows us to approach the universe as a membrane in higher dimension. The picture based on this approach conducts to a cyclic universe. In this picture the Universe undergoes an endless sequence of cycles in which it contracts in a big crunch and re-emerges in an expanding big bang which represents a bridge to a previous contracting era. The temperature and density of the universe do not become infinite at any point in the cycle and no singularities arise from this theory [1]. Yet, the cyclic model includes all of the theoretical predictions of the inflationary theory.

2. THE GEOMETRY OF A DOUGHNUT

The torus is familiar to us as the surface of a doughnut. We may describe it as a subspace of \mathcal{R}^3 geometrically. For example, a torus is obtained if we sweep a circle around an axis (Fig. 1a) or, through a diagram in which the opposite edges of a rectangle are glued (Fig. 1b).

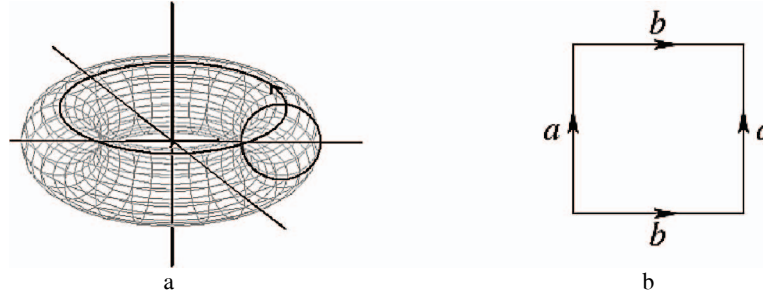


Fig. 1. – Two ways to construct a torus: by sweeping a circle around an axis (a) or by gluing the opposite edges of a rectangle (b).

Let us denote by α and β the radius of the tube and the radius from the torus centre to the center of the tube and by x_a , $a = 1 \dots 3$ the coordinates of a Cartesian system with O_{x_1} on the direction of the torus axis of symmetry (Fig. 2).

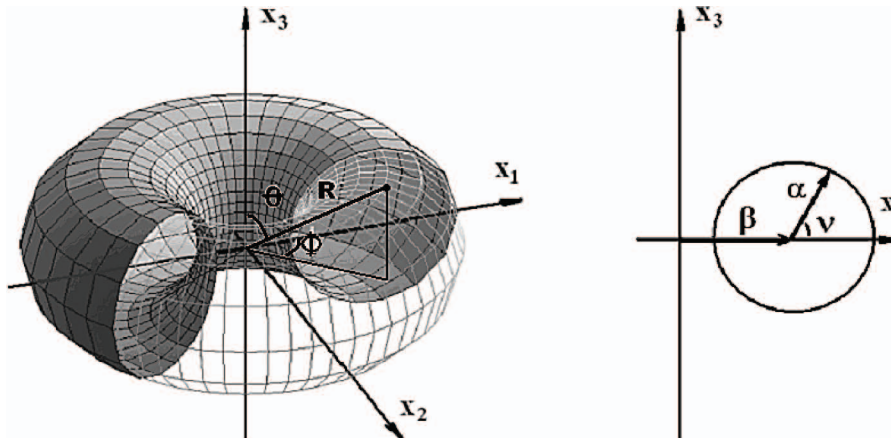


Fig. 2. – Coordinates used on the torus. The position of a point on the torus is given by the coordinates (R, θ, ϕ) .

Then, the surface equation in Cartesian coordinates is

$$\left(\beta - \sqrt{x_1^2 + x_2^2}\right)^2 + x_3^2 = \alpha^2 \quad (1)$$

and the parametric equations are

$$\begin{aligned} x_1 &= (\beta + \alpha \cos v) \cos \phi \\ x_2 &= (\beta + \alpha \cos v) \sin \phi \\ x_3 &= \alpha \sin v \end{aligned} \quad (2)$$

These equations give the Riemannian metric

$$ds^2 = (\beta + \alpha \cos v)^2 d\phi^2 + \alpha^2 dv^2 \quad (3)$$

Alternatively, the position of a point on the torus surface is specified in spherical coordinates by

$$\begin{aligned} x_1 &= R \sin \theta \cos \phi \\ x_2 &= R \sin \theta \sin \phi \\ x_3 &= R \cos \theta \end{aligned} \quad (4)$$

From (2) and (4) we find

$$R^2 - 2R\beta \sin \theta + \beta^2 - \alpha^2 = 0 \quad (5)$$

3. CYCLIC UNIVERSES

Let us consider now the Ekpyrotic Universe as it is described in ref. [1]: two three-dimensional branes collide, stick together and the transformation of kinetic energy into heat gives the conditions for our universe to born. Fig. 3 represents a sketch of our brane moving along the time and the evolution of a universe A, maximally symmetric, from Big Bang to Big Crunch.

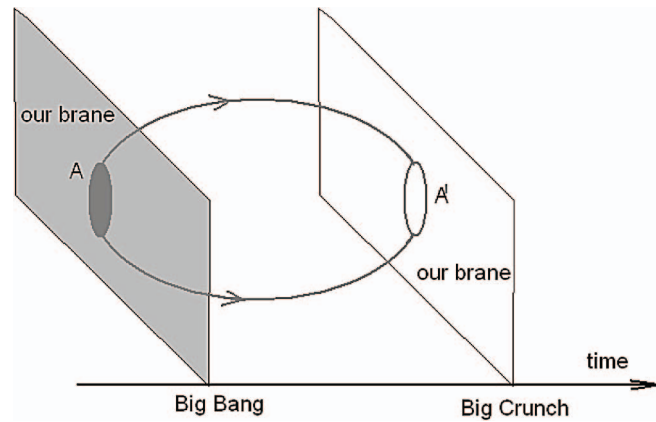


Fig. 3. – Our brane in the Ekpyrotic model. By identifying A with A' we obtain a torus.

The basics of the model are described in [2]. Note that the topology becomes that of a torus when the condition of cyclic universe $A \equiv A'$ is imposed. One can see that the universe can be viewed as a closed, tensioned string moving on the surface of a torus.

Reconsider now the equation (5). The usual coordinates on the string world-sheet surface (σ, τ) are given by

$$\begin{aligned} d\sigma &= R d\phi \equiv (\beta + \alpha \cos v) d\phi \\ d\tau &= \alpha dv \end{aligned} \quad (6)$$

It results that v and θ are time-like coordinates while ϕ is a space-like one. The equation (5) has the solutions

$$R_{1,2} = \beta \sin \theta \pm \sqrt{\beta^2 \sin^2 \theta - (\beta^2 - \alpha^2)} \quad (7)$$

and from these solution arise particular cases as follows:

- $\alpha = \beta$. We have the solutions $R_1 = 0$ and $R_2 = 2\beta \sin \theta$. The later solution is more interesting: it corresponds to a closed universe with a classical big-bang in the sense of an initial singularity, with a big crunch at $\theta = \pi$, with a maximum of expansion at $\theta = \pi/2$, where $R = 2\beta$. A horn torus illustrates this case (Fig. 4)

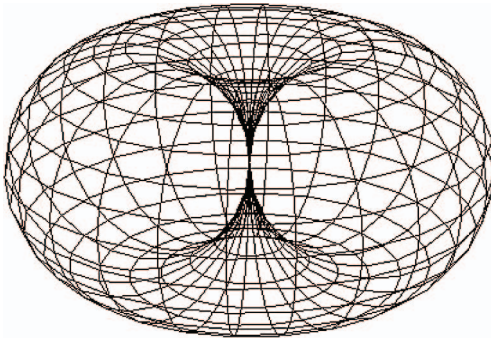


Fig. 4. – The horn torus shows an oscillating universe with Big Bang.

- $\alpha > \beta$. The solutions are $R_{1,2} = \beta \sin \theta \pm \beta \sqrt{\left(\frac{\alpha}{\beta}\right)^2 - \cos^2 \theta}$ and describe the evolution of an ekpyrotic, respectively a bouncing universe (Fig. 5).

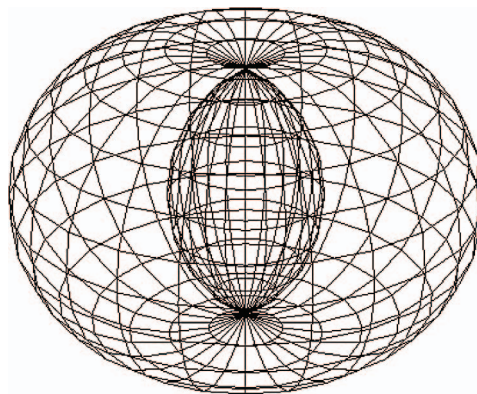
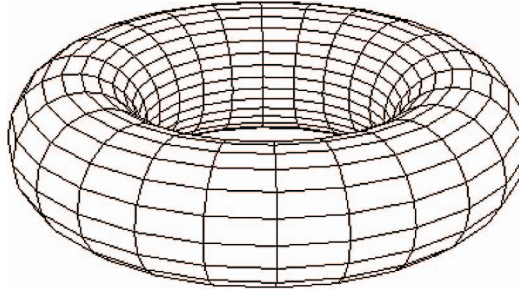


Fig. 5. – A spindle torus illustrates both the ekpyrotic and the bouncing universe.

- $\beta > \alpha$, $R_{1,2} = \beta \sin \theta \pm \sqrt{\beta^2 \sin^2 \theta + \alpha^2 - \beta^2}$ has an interesting limit at $\beta \gg \alpha$. The solution for R is, in this case, $R_{1,2} = \beta e^{\pm i\varphi}$ ($\varphi = \pi/2 - \theta$). It shows an oscillating universe which acts like a string which propagates on the torus.

Fig. 6. – The doughnut represents an oscillating universe without singularities.



4. DISCUSSION

The torus model of the space-time is a new approach of the cyclic universes which propose that the Universe runs endless from contraction to re-expanding state, from a Big Bang which is not necessarily a singularity to a Big Crunch. In the section above three extremely intuitive examples are given. The simplicity of the results emphasizes a natural way to describe the cyclic universes. In this model the space-time is shown to be rather flat than curved and includes periods of inflation. It does not give an answer to the question “why”, yet includes all of the successful results of the standard cosmology. Moreover, the geometry of the torus can suggest answers to the most of the still unsolved problems: the flatness, the horizon problem, the formation of structures and the problem of entropy.

REFERENCES

1. P. J. Steinhardt, N. Turok: arXiv: astro-ph/0204479 (2002)
2. J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D 64, (2001).