THE LANDAU AND LIFSHITZ PRESCRIPTION FOR A STRINGY MAGNETIC BLACK HOLE

IRINA RADINSCHI, BRINDUSA CIOBANU[*]

Department of Physics, "Gh. Asachi" Technical University,
Iasi, 700050, Romania

In this paper we calculate the energy distribution for two metrics which describe recently derived non-asymptotically flat black hole solutions in dilaton-Maxwell gravity and were given by Chan, Mann and Horne. The calculations are performed with the Landau and Lifshitz energy-momentum complex.

For the first metric the energy distribution depends on the mass M, charge Q of the black hole and γ parameter. For the second solution the energy depends on the mass M and charge Q of the black hole.

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1. INTRODUCTION

The subject of the localization of energy still lacks of a definite answer. For solving the problem many researchers have computed the energy as well as momentum and angular momentum associated with various space-times. The different attempts at constructing an energy-momentum density don't give a generally accepted expression. After the Einstein work [1]-[2], a plethora of energy-momentum complexes were constructed, including those of Einstein [1]-[2], Landau and Lifshitz [3], Papapetrou [4], Bergmann [5], Weinberg [6] and Møller [7]. These expressions, except that of Møller [7], were restricted to evaluate the energy distribution in quasi-Cartesian coordinates. Hence, an interesting question is whether any of the aforementioned prescriptions provides the best option for energy-momentum localization and, of course, under what conditions.

Virbhadra and his collaborators re-opened the problem of the energy and momentum localization and since then, many interesting results was obtained in this area [8]. Virbhadra established an important result showing that different energy-complexes (ELLPW) yield the same result for a general non-static spherically symmetric metric of the Kerr-Schild class [8]. Furthermore, these definitions (ELLPW) comply with the quasi-local mass definition of Penrose for a

[*] radinschi@yahoo.com, iradinsc@phys.tuiasi.ro, bciobanu2003@yahoo.com

general non-static spherically symmetric metric of the Kerr-Schild class. However, these prescriptions disagree in the case of the most general non-static spherically symmetric metric. According to Virbhadra, although the energy-momentum complexes behave under general coordinate transformations like non-tensorial objects, the local conservation laws obtained by them hold in all coordinate systems. Also, many attempts have been performed to establish a definition of quasilocal energy in General Relativity [9] and to construct superenergy tensors [10].

In this paper we move on towards analyzing the energy for two metrics which describe recently derived non-asymptotically flat black hole solutions in dilaton-Maxwell gravity and were given by Chan, Mann and Horne [15]. The calculations are performed with the Landau and Lifshitz energy-momentum complex and we use the Maple program which has attached the GrTensor II version 1.50 platform. We study the energy associated with this solution because we think it can furnish us interesting results and we extend the investigations related to the energy localization to several black hole solutions in the Einstein-dilaton-Maxwell theory. Through the paper we use geometrized units ($G = 1$, $c = 1$) and follow the convention that Latin indices run from 0 to 3.

2. ENERGY DISTRIBUTION IN THE LANDAU AND LIFSHITZ PRESCRIPTION

String theory may be the best way to attain the holy grail of fundamental physics, which is to generate all matter and forces of nature from one basic building block. We mention the words of David Gross: "String theory is not a smooth continuation of our previous attempts, of our previous improvements of the theory of matter. The basic idea of string theory is that all the particles, all the carriers of the forces, and, as it turns out, even the carriers of the forces of gravity, all are, in a sense, excitations of a string-like object. They appear to us, when we observe them at large distances, as particular modes of one fundamental string-like object. Now we say the string is fundamental. So there still are fundamental building blocks: they are the different modes of the string. One string, in some sense, corresponds to an infinite number of elementary particles, because each string can vibrate in an infinite number of ways. All of the higher harmonics of the string look like more massive excitations of different elementary particles. What is unifying and simple about a string is that all of those particles are all excitations of the one object". Through the years, many important studies have been made related to the string theory.

About the low energy effective theory we point out that largely resembles general relativity with some new "matter" fields as the dilaton, axion, etc. [11]-[12]. One of its main property is that there are two different frames in which the features of the space-time may look very different. These two frames are the Einstein frame and the string frame and they are related to each other by a conformal

transformation $\left(g_{\mu\nu}^{E} = e^{-2\phi} g_{\mu\nu}^{S}\right)$ which involves the massless dilaton field as the conformal factor. The string "sees" the string metric. Many of the important symmetries of string theory also rely of the string frame or the Einstein frame [13]. The T duality [13] transformation relates metrics in the string frame only, whereas S duality [14] is a valid symmetry only if the equations are written in the Einstein frame. Kar [11] gave important results about the stringy black holes and energy conditions. There were studied several black holes in two and four dimensions with regard to the Weak Energy Conditions (WEC). It is very important to study the black holes in string theory and this can be explained in the following way. Because string theory is expected to provide us with a finite and clearly defined theory of quantum gravity, the answer of many questions related with black hole evaporation could be solved in the context of string theory. To do this, is required to construct black hole solutions in string theory.

In the Einstein-dilaton-Maxwell theory the action is given by

$$S_{EDM} = \int d^4x \sqrt{-g}\, e^{-2\phi} \left[\mathsf{R} + 4 g_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi - \frac{1}{2} g^{\mu\nu} g^{\nu\rho} F_{\mu\nu} F_{\lambda\rho} \right]. \tag{1}$$

Varying with respect to the metric, dilaton and Maxwell fields we get the field equations for the theory given as

$$R_{\mu\nu} = -2\nabla_\mu \nabla_\nu \phi + 2 F_{\mu\lambda} F_\nu{}^\lambda, \tag{2}$$

$$\nabla^\nu \left(e^{-2\phi} F_{\mu\nu} \right) = 0 \tag{3}$$

$$4\nabla^2 \phi - 4 \left(\nabla\phi \right)^2 + R - F^2 = 0. \tag{4}$$

Recently, some non-asymptotically flat black hole solutions in dilaton-Maxwell gravity are due to Chan, Mann and Horne [15]. For the first solution, the metric in the Einstein frame is given by

$$ds^2 = \frac{C}{\gamma^4} dt^2 - \frac{1}{C} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2, \tag{5}$$

where $C = r^2 - 4\gamma^2 M$.

For the second black hole solution, the string metric is obtained by performing the usual conformal transformation on the metric. This turns out to be

$$ds^2 = \frac{r^2 D}{\gamma^4} dt^2 - \frac{1}{D} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2, \tag{6}$$

with $D = 1 - \dfrac{2\sqrt{2}\gamma^2 M}{Qr}$.

The string metric for the magnetic black hole is given by

$$ds^2 = \frac{2Q^2 E}{\gamma^4} dt^2 - \frac{2Q^2}{r^2 E} dr^2 - 2Q^2 r^2 d\theta^2 - 2Q^2 r^2 \sin^2 \theta d\varphi^2, \tag{7}$$

where $E = 1 - \dfrac{4M}{r}$.

We calculate the energy of these black hole solutions and study the dependence of the energy on the parameters involved in the expression of the metrics.

The Landau and Lifshitz energy-momentum complex [3] is given by

$$L^{ij} = \frac{1}{16\pi} S^{ijkl}{}_{,kl}, \tag{8}$$

where

$$S^{ijkl} = -g\left(g^{ij}g^{kl} - g^{ik}g^{jl}\right). \tag{9}$$

L^{00} and $L^{0\alpha}$ are the energy and, respectively, the momentum density components.

The Landau and Lifshitz energy-momentum complex satisfies the local conservation laws

$$\frac{\partial L^{ij}}{\partial x^j} = 0. \tag{10}$$

Integrating L^{ik} over the three-space gives the energy and momentum components

$$P^i = \iiint L^{i0} dx^1 dx^2 dx^3. \tag{11}$$

P^0 is the energy and P^α are the momentum components.

Using the Gauss theorem we obtain

$$P^i = \frac{1}{16\pi} \iint S^{i0\alpha} n_\alpha dS, \tag{12}$$

where $n_\alpha = \left(\dfrac{x}{r}, \dfrac{y}{r}, \dfrac{z}{r} \right)$ are the components of a normal vector over an infinitesimal

surface element $dS = r^2 \sin\theta \, d\theta \, d\varphi$.

For making the calculations using the Landau and Lifshitz prescription we transform the metrics given by (6) and (7) to quasi-Cartesian coordinates t, x, y, z according to $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$ and $z = r\cos\theta$.

The expression of the energy distribution for the metric given by (6) is

$$E(r) = \frac{r}{2} \frac{2\sqrt{2}\gamma^2 M}{\left(Qr - 2\sqrt{2}\gamma^2 M \right)}. \tag{13}$$

The energy distribution depends on the mass M, charge Q and γ parameter.

In the Fig.1 E is plotted against r on X-axis and Q on Y-axis. For the mass M and γ parameter we get the values $M = 1$ and $\gamma = 1$.
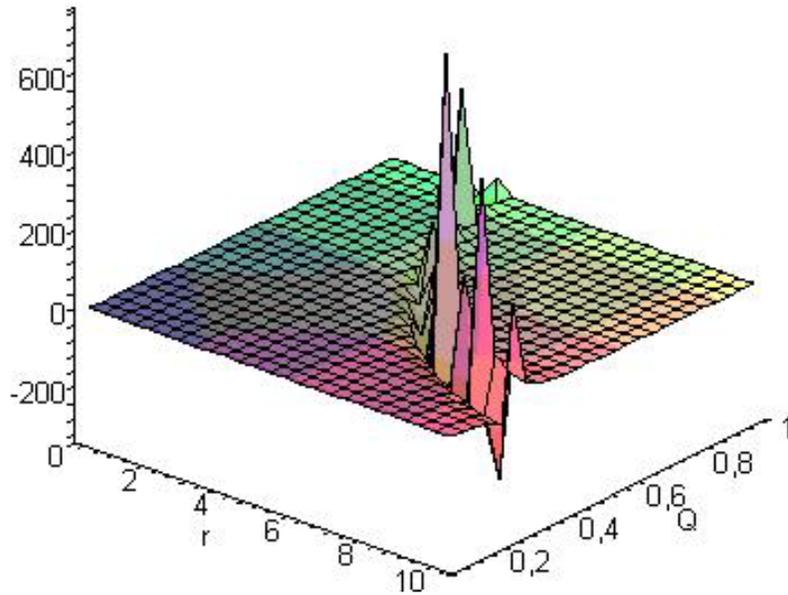

Fig.1

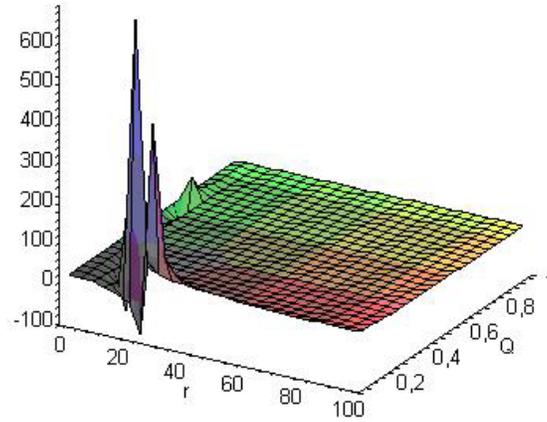Now, we choose other values for r and Q and we get the graph representation from Fig. 2.

Fig. 2

For the metric given by (7) we get for the energy.

$$E(r) = \frac{r}{2}\left(\frac{2Q^2}{r^2\left(1 - \dfrac{4M}{r}\right)} - 2Q^2 \right).$$ (14)

The energy distribution depends on the mass M and charge Q.
In the Fig. 3 E is plotted against r on X-axis and Q on Y-axis. The mass M takes the value $M = 1$.
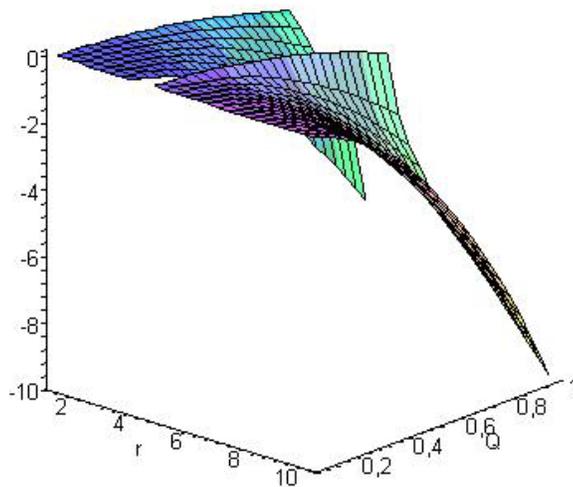

Fig. 3

We also consider that the mass M takes the value $M = 1$ and we have other for r and Q. In Fig. 4 E is plotted against r on X-axis and Q on Y-axis.
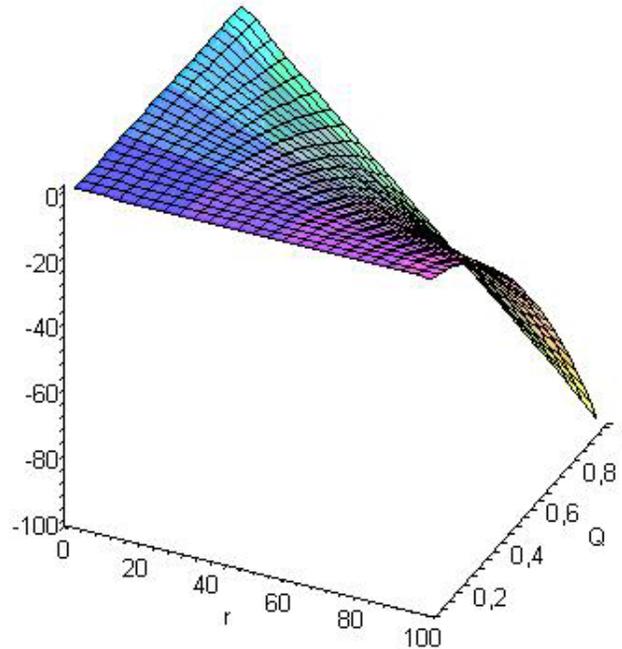

Fig. 4

3. DISCUSSION

At the same time with the Virbhadra pioneer's works [8], Bondi [16] gave his opinion that "a nonlocalizable form of energy is not admissible in general relativity, because any form of energy contributes to gravitation and so its location can in principle be found". Misner et al. [17] sustained that to look for a local energy-momentum means that is looking for the right answer to the wrong question. Also, they concluded that the energy is localizable only for spherical systems. On the other hand, Cooperstock and Sarracino [18] demonstrated that if the energy is localizable in spherical systems then it is also localizable in any space-times. Also, Chang, Nester and Chen [19] showed that the energy-momentum complexes are actually quasilocal and legitimate expression for the energy-momentum. They concluded that there exist a direct relationship between energy-momentum complexes and quasilocal expressions because every energy-momentum complexes is associated with a legitimate Hamiltonian boundary term. As we pointed out

above, the subject of the localization of energy continues to be an open one since Einstein has given his significant result of the special theory of relativity that mass is equivalent to energy. About the energy and momentum localization using several energy-momentum complexes we remark that it has many adepts but there was, also, many criticism related to the use of these prescriptions. The main lack of the energy-momentum complexes is that most of these restrict one to calculate in quasi-Cartesian coordinates.

Consequently, in this work we have implemented Landau and Lifshitz's formalism to calculate the energy distributions associated with two metrics which describe recently derived non-asymptotically flat black hole solutions in dilaton-Maxwell gravity and were given by Chan, Mann and Horne [15]. We perform the calculations with the Maple program which has attached the GrTensor II version 1.50 platform [20]-[21]. We obtained acceptable (meaningful) expressions for the energy density distribution. For the first black hole solution the energy distribution depends on the mass M, charge Q and γ parameter. In the second case, the energy depends on the mass M and charge Q. Also, we plot in some particular cases, these energy distributions. We evaluate the energy distribution in Schwarzschild Cartesian coordinates. In a future work, we will compare these results with those obtained in Kerr-Schild Cartesian coordinates in the Einstein, Landau and Lifshitz, Papapetrou and Weinberg prescriptions.

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