DYNAMICS OF A SUPER RADIANT DISSIPATIVE SYSTEM OF ELECTRONS TUNNELING IN A MICRO-CAVITY

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We discuss the super-radiant dissipative tunneling of electrons in a perfectly tuned micro-cavity of a p-i-n semiconductor structure with quantum dots. Our description is based on a quantum master equation with microscopic coefficients, depending on two-body potentials, densities of the dissipative environment states, and temperature, that is in accordance with the detailed balance principle. For this system, we obtain Maxwell-Bloch equations with explicit microscopic dissipative coefficients, while taking into account a current that could be injected in the device, and the field dissipation and propagation, that essentially determine the super-radiation process. We are especially interested in the absolute values of the super-radiant pulse amplitude and in the space and time-distributions as functions of physical characteristics and operation conditions of the system. Due to the planar distribution of the quantum dots, at a low density of these dots the super-radiant exponent is 3, not 2 as it is for a volume distribution. Due to Rabi oscillations, in an under-damped system the super-radiant exponent decreases with the quantum dot density, tending to 1.5. Power densities of the order of those that could be absorbed from Sun at the level of our planet are easily obtained for realistic values of the system parameters and operation conditions.

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Despite a long history, the atom-field interaction is still an active field of investigation especially due to the dissipative processes [1], standing at the basis of important technical applications – an example is a new amplifying device based on the dissipative super-radiant tunneling, recently proposed by M. Asada [2]. The super radiance of a system of electrons predicted by Dike [3] has been intensively studied taking into account various physical effects as: (1) the statistical distribution of the electron states [4], (2) level degeneracy effects [5, 6], (3) Langevin forces acting on the atomic system [7], (4) transverse effects [8], (5) competing of three-photon and one-photon transitions [9], (6) the super radiance spectrum [10], (7) existence of photon gaps [11–14], (8) spontaneously generated coherence effects [15], (9) super radiance suppression by scattering

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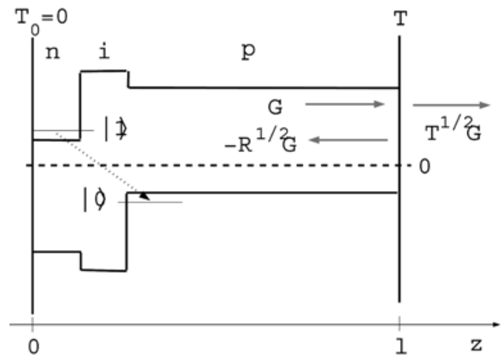
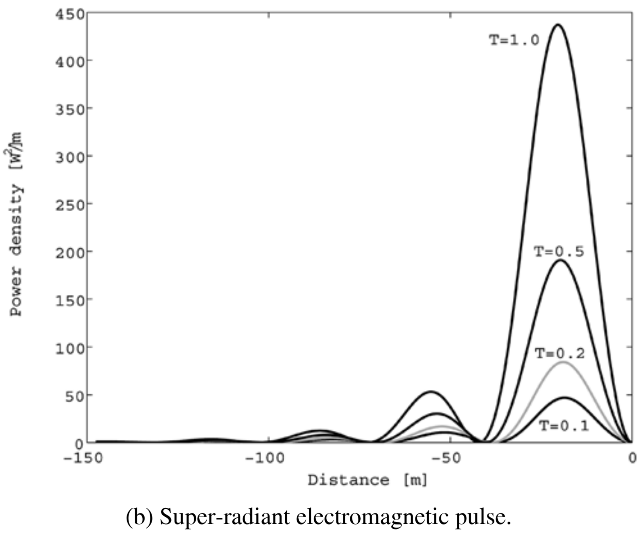
[16]. However, a detailed study of the super-radiant power as a function of the main physical characteristics of a specific system do not yet exist in literature, and the accordance of the dissipative super-radiant dynamics with the detailed balance principle [17–19] has not yet been discussed.

We investigate the super-radiance of a p-i-n semiconductor structure with quantum dots and a perfectly tuned micro-cavity (Fig. 1). Here the electrons decaying between well-determined energy levels on one side of this cavity build-up a super-radiant field transmitted out on the other side. The insulating region i enables a thorough control of the transition dipole $r_{01} \equiv \left|\langle 0 | \vec{r} \,| 1 \rangle\right|$ that determines the matter-field coupling. We describe this system by a Hamiltonian

$$H = H^S + H^F + V, \tag{1}$$



(a) Super-radiant semiconductor structure.



(b) Super-radiant electromagnetic pulse.

Fig. 1. – Super-radiant decay in a quantum dot p-i-n structure micro cavity.

with three terms: $H^S = \sum_i \varepsilon_i c_i^+ c_i$, $i = 0, 1$ for the system of electrons, $H^F = \hbar\omega\left(a^+ a + \frac{1}{2}\right)$ for the super-radiant field mode of frequency $\omega = \omega_{10}$, and the interaction potential

$$V = ie \sum_{ij} \omega_{ij} r_{ij} c_i^+ c_j A. \tag{2}$$

depending on the transition frequencies ω_{ij}, on the dipole moments r_{ij}, and on the potential vector

$$A = \frac{\hbar}{e} K \left(a e^{i\vec{k}\vec{r}} + a^+ e^{-i\vec{k}\vec{r}} \right), \tag{3}$$

where $K = \sqrt{\alpha_0 \frac{\lambda}{V}}$, while $\alpha_0 = \frac{e^2}{4\pi\varepsilon\hbar c}$, λ is the wavelength, and $\mathcal{V}$ is the quantization volume. We also take into account the existence of a complex dissipative coupling of the system described by the Hamiltonian H with (1) other electrons in the profound clusters of the n-region, (2) the crystalline lattice, (3) the free electromagnetic field coupled with the system of electrons, and (4) other dissipative elements of the semiconductor structure coupled with the super-radiant mode, that essentially determines the radiation process. The description of this process depends on the system-environment interaction model and on the approximations used to reduce a quantum dynamical equation to a master equation [20–23].

In this paper, we consider an explicit quantum master equation, with two-body potentials between the system and environment particles as describing the most probable dissipative processes, of single-particle transitions of the system $c_i^+ c_j$ correlated with single-particle transitions of the environment [19, 24]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \sum_{i,j=1}^{N} \lambda_{ij}([c_i^+ c_j \rho(t), c_j^+ c_i] + [c_i^+ c_j, \rho(t) c_j^+ c_i]). \tag{4}$$

This equation for an N-level system has $N^2 - 1$ explicit dissipative coefficients

$$\lambda_{ij} = \lambda_{ij}^F + \lambda_{ij}^B + \gamma_{ij}, \tag{5}$$

with terms describing decay/excitation processes that for transition energies much higher than temperature, $\varepsilon_{ji} \gg T$, $j > i$, have the approximate expressions:

$$\lambda_{ij}^F = \frac{\pi}{\hbar} \left| \langle \alpha i | V^F | \beta j \rangle \right|^2 \left[1 - f_\alpha^F(\varepsilon_{ji}) \right] g_\alpha^F(\varepsilon_{ji}) \tag{6a}$$

$$\lambda_{ji}^{F} = \frac{\pi}{\hbar}\left|\langle \alpha i | V^{F} | \beta j \rangle\right|^{2} f_{\alpha}^{F}(\varepsilon_{ji}) g_{\alpha}^{F}(\varepsilon_{ji}), \tag{6b}$$

for a dissipative environment of Fermions,

$$\lambda_{ij}^{B} = \frac{\pi}{\hbar}\left|\langle \alpha i | V^{F} | \beta j \rangle\right|^{2} \left[1 + f_{\alpha}^{B}(\varepsilon_{ji})\right] g_{\alpha}^{B}(\varepsilon_{ji}) \tag{7a}$$

$$\lambda_{ji}^{B} = \frac{\pi}{\hbar}\left|\langle \alpha i | V^{F} | \beta j \rangle\right|^{2} f_{\alpha}^{B}(\varepsilon_{ji}) g_{\alpha}^{B}(\varepsilon_{ji}) \tag{7b}$$

for a dissipative environment of Bosons, and

$$\gamma_{ij} = \frac{2\alpha_{0}}{c^{2}\hbar^{3}} \vec{r}_{ij}^{\,2} \varepsilon_{ji}^{3} \left(1 + \frac{1}{e^{\varepsilon_{ji}/\mathcal{T}} - 1}\right) \tag{8}$$

for the free electromagnetic field. In these expressions, V^F, V^B are dissipative two-body potentials, $g_{\alpha}^{F}(\varepsilon_{ji})$, $g_{\alpha}^{B}(\varepsilon_{ji})$ are densities of the environment states, and $f_{\alpha}^{F}(\varepsilon_{ji})$, $f_{\alpha}^{B}(\varepsilon_{ji})$ are the Fermi-Dirac and respectively Bose-Einstein distributions. We remark that these coefficients satisfy the detailed balance principle for a dissipative environment of Fermions [19]:

$$\frac{\lambda_{ji}^{F}}{\lambda_{ij}^{F}} = \frac{f_{\alpha}^{F}(\varepsilon_{ji})}{1 - f_{\alpha}^{F}(\varepsilon_{ji})} = e^{-\varepsilon_{ji}/\mathcal{T}}, \tag{9}$$

and of Bosons, or, particularly, of the free electromagnetic field

$$\frac{\lambda_{ji}^{B}}{\lambda_{ij}^{B}} = \frac{f_{\alpha}^{B}(\varepsilon_{ji})}{1 + f_{\alpha}^{B}(\varepsilon_{ji})} = e^{-\varepsilon_{ji}/\mathcal{T}}. \tag{10}$$

That means that in fact dissipation is a necessary part of the quantum dynamics, otherwise this principle making no sense. Considering a two-level system, $i, j = 0, 1$ with negligible dimensions in comparison with the field wave length, and a perfectly tuned cavity, from (4) with the Hamiltonian (1) one obtains the Maxwell-Bloch equations

$$\frac{\mathrm{d}}{\mathrm{d}t} u(t) = -\gamma_{\perp} u(t) - gG(0,t)w(t) \tag{11a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} w(t) = -\gamma_{\|}[w(t) - w_{0}] + 2\Phi + (2 - T)gG(0,t)u(t) \tag{11b}$$

$$\frac{\partial}{\partial t} G(z,t)\Big|_{z=0} - \sqrt{T}c\frac{\partial}{\partial z} G(z,t)\Big|_{z=0} = -\gamma_{EM} G(z,t)\Big|_{z=0} - g\frac{\hbar\omega}{4\varepsilon\mathcal{V}} u(t) \tag{11c}$$

$$\frac{\partial}{\partial t} G(z,t) + c\frac{\partial}{\partial z} G(z,t) = 0, \quad z > 0 \tag{11d}$$

with the polarization $u(t)$, the population $w(t)$, the electromagnetic field amplitude $G(z,t)$, a particle flux Φ that could be injected in the device, the matter-field coupling coefficient $g = \frac{e}{\hbar} r_{01}$, the decay rate of the super-radiant mode γ_{EM}, and the dissipative coefficients

$$\gamma_{\perp} = \lambda_{01} + \lambda_{10} + \lambda_{00} + \lambda_{11} \tag{12a}$$

$$\gamma_{\parallel} = 2(\lambda_{01} + \lambda_{10}) \tag{12b}$$

$$w_0 = -\frac{1 - e^{-\hbar\omega_0/\mathcal{T}}}{1 + e^{-\hbar\omega_0/\mathcal{T}}}, \tag{12c}$$

where $\mathcal{T}$ is temperature. The second coefficient of the last term of Eq. (11b) takes into account the coupling with the two counter propagating waves in the cavity, while $\mathcal{T}$ describes the decrease of the matter-field coupling due to the field radiation through the output mirror. From Eq. (11c) we notice that the polarization $u(t)$ in the quantization volume $\mathcal{V} = 1/N_e^{3/2}$, N_e being the number of quantum dots on the area unit, is a source for two field variations: (1) a time-variation of the field in the cavity $G(z,t)\big|_{z=0}$, and (2) a field flow through the mirror with the transmission coefficient T, where the transmitted field $G_T(z,t) = \sqrt{T} G(z,t)$ propagates according to Eq. (11d). For $T = 0$ Eqs. (11) describe a closed cavity, while for $T = 1$ they correspond to an open super radiant structure. When the dissipative coefficients are neglected, these equations satisfy conservation relations:

$$(2-T)V\frac{\mathrm{d}}{\mathrm{d}t}W(0,t) + \hbar\omega\frac{\mathrm{d}}{\mathrm{d}t}\rho_{11}(t) = 0 \qquad \text{(energy)} \tag{13a}$$

$$(2-T)u^2(t) + w^2(t) = 1 \qquad \text{(Bloch vector)}, \tag{13b}$$

where $W(0,t) = \varepsilon G^2(0,t)$ is the energy density of the electromagnetic field, while $\rho_{11}(t) = \frac{1}{2}[1 + w(t)]$ is the population of the upper level. For $T = 0$, these relations describe a closed cavity, with a factor 2 for the two counter propagating waves, while for $T = 1$ they correspond to an open structure, with a single radiation mode. In comparison with other Maxwell-Bloch equations used in the super-radiance domain as Eqs. (47), (48), (49) in [7], our equations (11) have explicit expressions of the dissipative coefficients (12a), (12b) with (5), (6), (7), (8), that satisfy the detailed balance conditions (9), (10). More than that, in (11c) we consider a current injection $I = eN_e\Phi$, and a dissipation of the super-radiant field, that, as it will be shown in the following, is necessary for the solution convergence.

In the following, we take a resonance energy $\hbar\omega_{10} = 0.1$ eV, and calculate the power density (Poynting vector amplitude) $S(z,t) = TS_0(z,t)$ with $S_0(z,t) = = cW(z,t) = c\varepsilon G^2(z,t)$ from Eqs. (11) with coefficients containing the main physical parameters of the system. From (12b) with (5–8), the decay rate gets a temperature-independent term and a temperature-dependent one, coming from the coupling of the system to the Fermion and respectively to the Boson part of the environment:

$$\gamma_{\|} = \gamma_{\|}^F + \gamma_{\|}^B \cdot \frac{e^{\hbar\omega_0/\mathcal{T}} + 1}{e^{\hbar\omega_0/\mathcal{T}} - 1}, \tag{14}$$

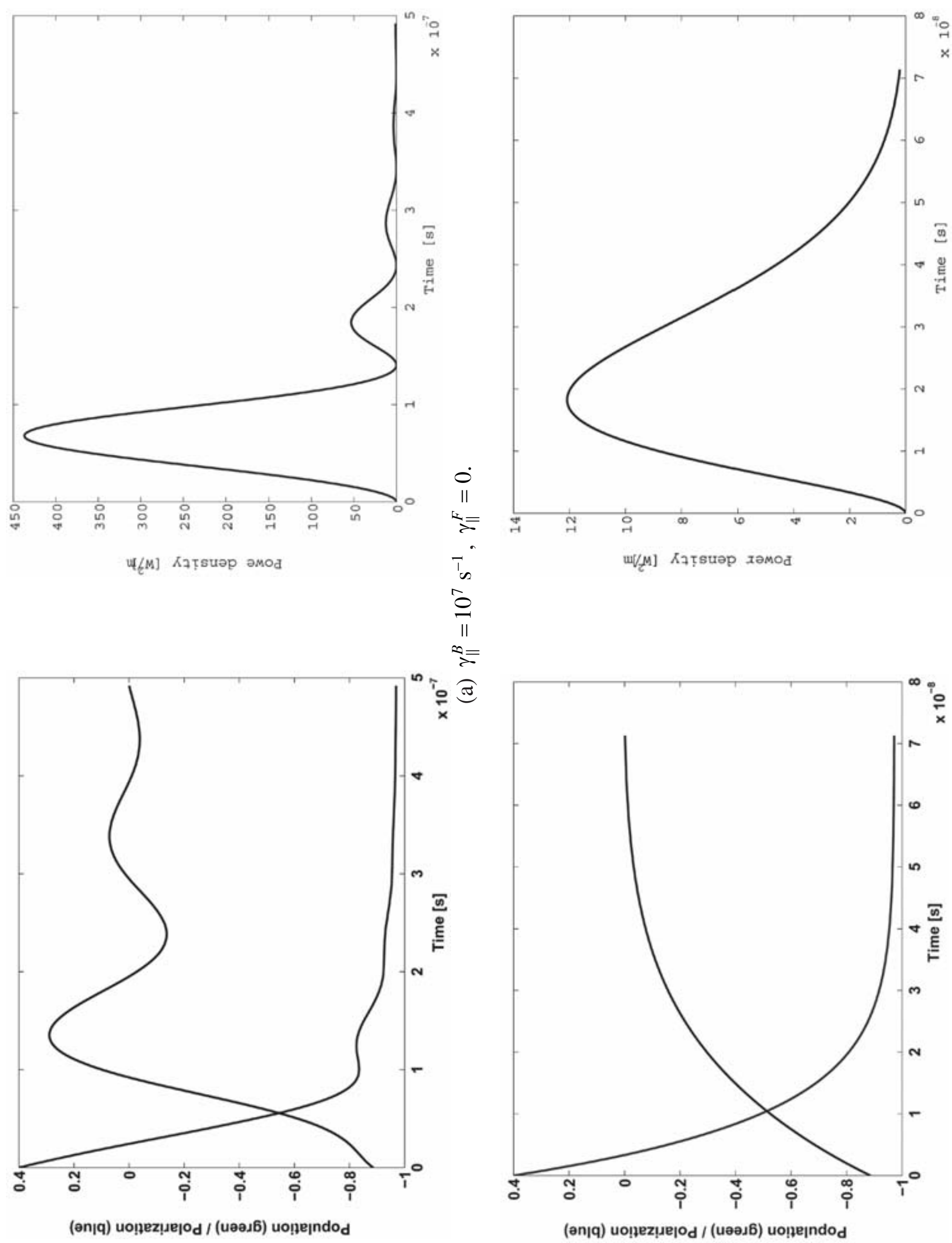
that, essentially, means a temperature dependence given by the detailed balance relations (9), (10). For simplicity, we take into account only the Boson component of the decay rate that depends on temperature, and consider $\gamma_\perp = \gamma_{\|}/2 = \gamma_{EM}$. In Fig. 2 we represent the time evolution of an open structure ($T = 1$) for two cases: (a) under-damped, and (b) over-damped, while in Fig. 1 (b) the power density is represented in space, for the under-damped case with the same parameters but different values of the transmission coefficient T. In these representations, we considered the initial condition of a thermal state $w(0) = 0.4$, $\mathcal{T} = 273.15$ K, $\Phi = 0$, and an initial polarization $|u(0)| = \sqrt{w_0^2 - w^2(0)}$ in accordance with the equilibrium condition: $w(0) \to w_0 \;\Rightarrow\; u(0) \to 0$.

Fig. 3 illustrates other two effects decreasing the super-radiation field amplitude: (1) the propagation of this field in an open structure, that diminishes the matter-field coupling, and (2) the field mode dissipation. From Fig. 3 (a), we notice that for the tunneling of a packet of N_e electrons, the field propagation and dissipation have somehow similar effects, diminishing the pulse amplitude without changing its shape (green and magenta curves in comparison with the blue curve). However, from Fig. 3 (b) we notice that when a current I is injected in the structure, the two terms of the field equation, of propagation and of dissipation, have qualitatively different effects. Only through the field dissipation term (the magenta curve – $T = 0$, or the red curve – $T = 1$), Eqs. (11) have a finite asymptotic solution of the electromagnetic field density of energy

$$W(\infty) \equiv \varepsilon G^2(0,\infty) = \frac{1}{2-T} \cdot \left[\frac{\hbar\omega}{4\gamma_{EM}} \left(2\frac{I}{e} N_e^{1/2} + w_0 \gamma_{\|} N_e^{3/2} \right) - \varepsilon \frac{\gamma_\perp \gamma_{\|}}{g^2} \right], \tag{15}$$

while the population is

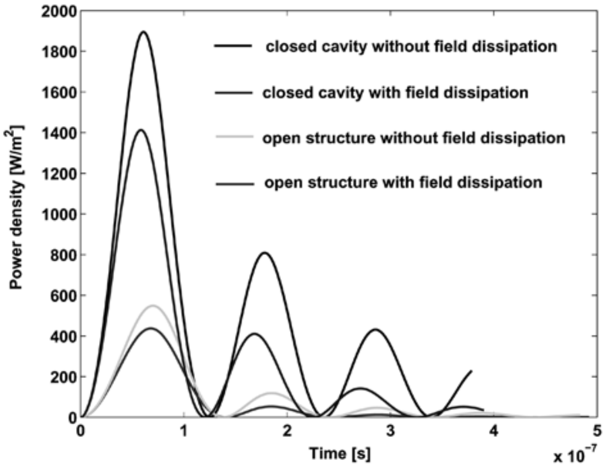
$$w(\infty) = \frac{w_0 + \dfrac{2I}{eN_e\gamma_{\|}}}{1 + (2-T)\dfrac{g^2}{\gamma_\perp\gamma_{\|}} G^2(0,\infty)}. \tag{16}$$



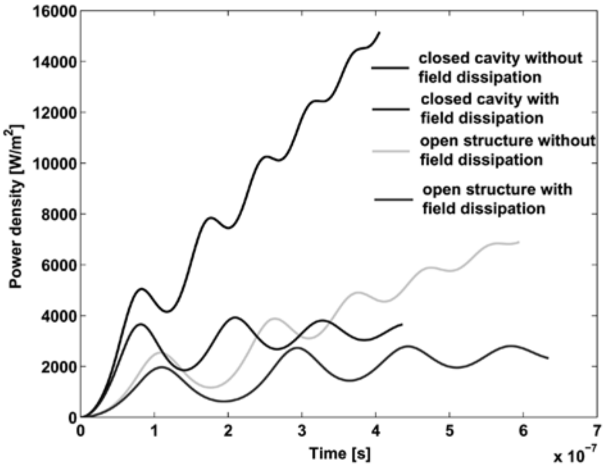
(a) $\gamma_{\parallel}^{B} = 10^{7}\ \mathrm{s}^{-1}$, $\gamma_{\parallel}^{F} = 0$.

(b) $\gamma_{\parallel}^{B} = 10^{8}\ \mathrm{s}^{-1}$, $\gamma_{\parallel}^{F} = 0$.

Fig. 2. – The population/polarization decay generating a super-radiant pulse in an open structure ($T = 1$), for an initial population $w(0) = 0.4$, a number of electrons (density of quantum dots) $N_e = 10^{16}$ electrons/m², a dipole moment $r_{01} = 10^{-6}$ nm and two values of the decay rate $\gamma_{\parallel}^{B}$ while $\mathcal{T} = 273.15$ K.



(a) Quantum tunneling without injected current ($\Phi = 0$).

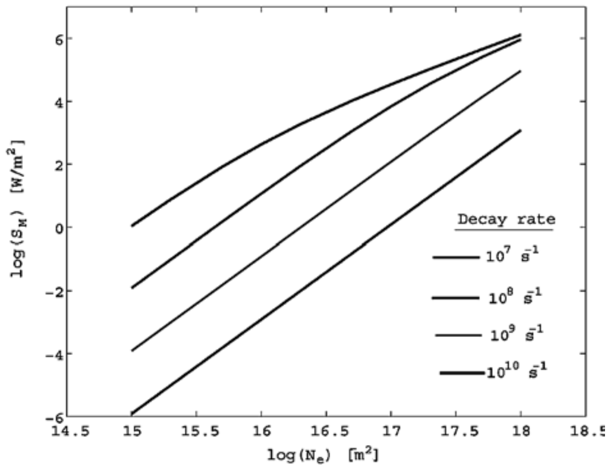


(b) Quantum tunneling with an injected current $I = eN_e\Phi = 20$ mA/mm^2.

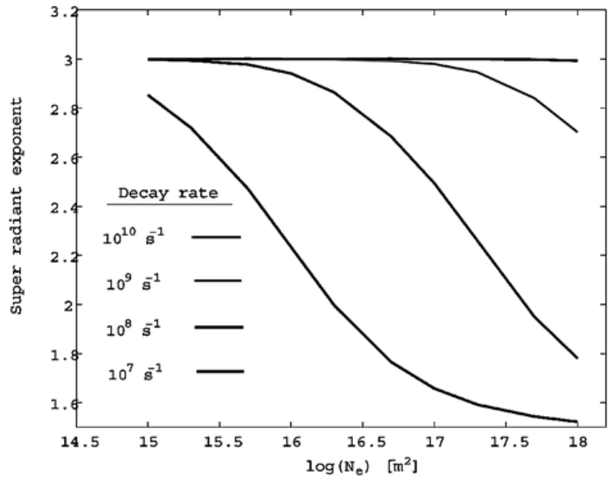
Fig. 3. – Super radiant pulse generation in quantum tunneling with and without injected current, for $w(0) = 0.4$, $N_e = 10^{16}$ quantum dots/m^2, $r_{01} = 10^{-6}$ nm, and $\gamma_{\|}^{B} = 10^7$ s^{-1}, $\gamma_{\|}^{F} = 0$.

Otherwise, the continuous polarization created by the injected current I determines a continuous increase of the field (the blue curve – $T = 0$, and the green curve – $T = 1$), that is a non-physical solution – this increase is limited by field dissipation.Without a field dissipation $(\gamma_{EM} = 0)$, for an injected current above a threshold, the density of energy (15) becomes ∞, while the population difference (16) becomes 0, that means a violation of the detailed balance principle, that requires a population inversion for canceling the difference between the environment-assisted decays and excitations.

The basic problem discussed from the beginning of the super-radiance domain [3], and that continues to be of interest [14], is the exponential dependence of the super-radiant pulse amplitude on the number of tunneling electrons (no injected current – $I = 0$). In Fig. 4 we represent this dependence for



(a) Absolute values.



(b) Super-radiant exponent.

Fig. 4. – The N_e-dependence of the super-radiant pulse amplitude S_M for different values of decay rate for a dipole moment $r_{01} = 10^{-6}$ nm, $N_e = 10^{16}$ quantum dots/m^2, and $w(0) = 0.4$.

different decay rates. For a strongly dissipative case, the super-radiant exponent is approximately 3, while for a weekly dissipative case, this exponent decreases due to the Rabi oscillation that tends to limit the amplitude of the super-radiant pulse when the frequency of this oscillation increases with the quantum dot density.

As a conclusion, from a quantum master equation entirely satisfying the quantum-mechanical and detailed balance principles we derived Maxwell-Bloch equations for a super-radiant structure with quantum dots and a perfectly tuned cavity. In comparison with other Maxwell-Bloch equations previously used in the super-radiance domain, these equations have explicit microscopic coefficients, with a temperature dependence according to the detailed balance conditions, and include an injected current and a dissipation of the field that is necessary for their internal consistency. We investigated the dissipative super-radiant tunneling having in view principal characteristics that determine the super-radiant power. For realistic values of the system parameters we obtained super-radiant power densities comparable to the power density provided by Sun at the level of our planet, that is approximately 2 kW/m^2, thus suggesting the application to an efficient transformation of the solar power to micro-waves, while this efficiency is monitored in the framework of a microscopic model.

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