# DYNAMIC LOCALIZATION EFFECTS UNDER TIME DEPENDENT ELECTRIC FIELDS

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*Received December 10, 2004*

An alternative approach to the dynamic localization of a charged particle moving on an one-dymensional lattice under the influence of a time dependent electric field is proposed. The present dynamic localization proceeds to leading order in terms of the large order zeros of the Bessel function $J_0(z)$, such as derived before by Dunlap and Kenkre, where $z$ stands for the ratio between the field magnitude and its frequency. Now one resorts to admissible time values, which results in a time discretization working in conjunction with the the related strong-field dynamic localization condition.

## 1. INTRODUCTION

The motion of charged particles in spatially periodic structures driven by time-dependent electric fields has received much attention [1–5]. Exact calculations concerning such motions have also been done for discrete lattices [6, 7]. In these latter cases it has been found that there is a periodic return of the particle to the initially occupied site when the ratio of the magnitude of the field to its frequency approaches selected values only. A such behavior is synonymous to the onset of dynamic localization. Relationships with Bloch-oscillations have been discussed, too [7, 8]. It has also been assumed that only electric fields exhibiting discontinuous sign-changes are able to produce an exact dynamic localization [9], but such rather surprising results require further clarifications. Recent experimental developments such as free-electron lasers [10], coupled optical waveguides [11], or tunable THz-radiation, led to a renewed interest in dynamic localization effects. For this purpose we shall account, for an alternative wavefunction satisfying exactly the Schrödinger equation by selecting admissible but discretized time values. Accordingly one deals with an exact solution proceeding under additional constraints. Choosing the cosinusoidal modulation, results in a dynamic localization condition which reproduces the one established before by Dunlap and Kenkre [6] within the strong field limit. Moreover, proceeding in this manner opens the way to derive dynamic localization conditions for arbitrary field modulations, which represents a useful finding.

## 2. PRELIMINARIES AND NOTATIONS

The system one deals with is described by the tight binding Hamiltonian

$$\mathcal{H}(t) = V \sum_{m=-\infty}^{+\infty} \left( |m\rangle\langle m+1| + |m+1\rangle\langle m| \right) - eE(t)a \sum_{m=-\infty}^{+\infty} m|m\rangle\langle m|, \tag{1}$$

where $V$ is the hopping parameter, while $a$ denotes the lattice spacing. The site number and the corresponding Wannier states are denoted by $m$ and $|m\rangle$, respectively. The time dependent electric field $E(t)$ is directed along the one dimensional lattice. We shall use the shorthand quotation

$$eaE(t) = E_F f(t), \tag{2}$$

where $e$ is the particle charge, $E_F$ stands for the field magnitude, whereas $f(t)$ expresses a periodic modulation. For convenience we shall assume that $E_F > 0$. Units for which $\hbar = a = 1$ are used. Resorting to a time dependent superposition of Wannier states yields the time-dependent discrete Schrödinger equation

$$i\frac{\mathrm{d}}{\mathrm{d}t}C_m(t) = V\left(C_{m+1}(t) + C_{m-1}(t)\right) - E_F m f(t) C_m(t), \tag{3}$$

where $C_m(t)$ denotes the expansion amplitude. The field free solution exhibits the closed form [12]

$$C_m^{(0)}(t) = \exp\left(-im\frac{\pi}{2}\right) J_m(2Vt) \tag{4}$$

where $J_m(z)$ is the Bessel-function of first kind and of order $m$ [13]. For this purpose the Jacobi-Anger formula (see (7.36), chapter 7.2, in [13])

$$\exp(iz\sin\varphi) = \sum_{n=-\infty}^{\infty} J_n(z)\exp(in\varphi), \tag{5}$$

has been used in conjunction with the momentum representation. Some other well known but useful formulae like

$$2\frac{\mathrm{d}}{\mathrm{d}z}J_m(z) = J_{m-1}(z) - J_{m+1}(z) \tag{6}$$

and $J_m(z) = (-1)^m J_{-m}(z)$ have also be recalled. One remarks that (4) obeys the initial condition

$$C_m^{(0)}(0) = \delta_{m,0} \tag{7}$$

which means that the initially occupied site is $m = 0$.

### 3. THE ALTERNATIVE WAVEFUNCTION

Looking for an alternative solution to (3) let us start from the Ansatz

$$C_m(t) = \exp\left(im\left(E_F\eta(t) - \frac{\pi}{2}\right)\right) J_n\left(\frac{2V}{E_F}u(t)\right), \tag{8}$$

where $u = u(t)$ is a time dependent function which remains to be specified later. One has

$$\eta(t) = \int_0^t f(t')\mathrm{d}t', \tag{9}$$

so that $C_m(t)$ obeys the initial condition $C_m(0) = \delta_{m,0}$ if $u(0) = 0$. Inserting (8) into (3) gives

$$\frac{\mathrm{d}u}{\mathrm{d}t} = E_F \cos(E_F\eta(t)), \tag{10}$$

provided that

$$\sin(E_F\eta(t)) = 0. \tag{11}$$

Equation (11) shows that we have to proceed further by selecting admissible time values. This results in the implementation of a time-grid, say $t = t_q$, in which case

$$E_F\eta(t_q) = n\pi + \varepsilon, \tag{12}$$

where $\varepsilon \to 0$ and where $n$ is an integer. Next, integrating (10) yields the solution

$$u(t) = E_F \int_0^t \cos(E_F\eta(t'))\mathrm{d}t', \tag{13}$$

which produces the limit $u(t) \to E_F t$ when $E_F \to 0$, as one might expect.

In particular, one would have

$$\eta(t) = \frac{1}{\omega}\sin(\omega t), \tag{14}$$

and

$$u(t) = E_F t J_0(z) + \frac{E_F}{\omega} \sum_{k=1}^{\infty} \sin(2k\omega t) \frac{J_{2k}(z)}{k}, \tag{15}$$

for a cosinusoidal modulation like

$$f(t) = \cos(\omega t), \tag{16}$$

where $z = E_F/\omega$. First we have to realize that the first term in the r.h.s. of (15), which is linear in *t*, is responsible specifically for the onset of delocalization effects, unless one considers that $J_0(z) = 0$ [6, 14]. On the other hand (12) becomes

$$\frac{E_F}{\omega} \sin(\omega t_q) = n\pi, \tag{17}$$

which relies on the larger order zeros of $J_0(z)$. Indeed, such zeros behave as

$$\frac{E_F}{\omega} = z = z_n \cong n\pi + O\left(\frac{1}{n}\right). \tag{18}$$

Then (17) becomes

$$\sin(\omega t_q) \cong 1, \tag{19}$$

while

$$\frac{E_F}{\omega} = n\pi, \tag{20}$$

stands for the pertinent dynamic localization condition. So it is clear that (20) reproduces the dynamic localization condition $J_0(E_F/\omega) = 0$ presented before [6], now in terms of large order zeros of $J_0(z)$. One would then have $J_0(z_n) \cong 0,$ where by now $|n| \gg 1.$ Moreover, there is

$$t_q = \frac{\pi}{2\omega}(4n' + 1), \tag{21}$$

by virtue of (19), where $n'$ is a further integer. Accordingly

$$\cos(\omega t_q) = 0, \tag{22}$$

which stands effectively for a field free limit. This means in turn that $\sin(2k\omega t) = 0$ where $k \in [1, \infty).$ So we are in a position to say that

$$u(t_q) \cong 0, \tag{23}$$

so that

$$\left|C_m\left(t_q\right)\right| \cong \left|C_m\left(0\right)\right|, \tag{24}$$

which proceeds in conjunction with (18). This means that the nodal points of the time grid just referred to above can also be viewed as being responsible for the return times to the initially occupied site.

The mean square displacement [6]

$$\left\langle m^2 \right\rangle = \sum_{m=-\infty}^{+\infty} m^2 \left|C_m^2\left(t\right)\right|, \tag{25}$$

exhibits the form

$$\left\langle m^2 \right\rangle = \frac{V^2}{E_F^2} u^2\left(t\right), \tag{26}$$

in accord with (8). Inserting $t = t_g$ one sees immediatey that $\left\langle m^2 \right\rangle$ vanishes to leading order by virtue of (18) and (23), which reflects a rather strong-localization of the particle on the nodal points of the time grid.

## 4. FURTHER APPLICATIONS

The dynamic localization condition (20) has been derived by eliminating the $t_q$ parameter between (17) and (22). This result can be generalized towards arbitrary $f\left(t\right)$-modulations by ruling out $t_q$ between the generalized counterpart of (22):

$$f\left(t_q\right) = 0, \tag{27}$$

and (12). Choosing *e.g.*, the sinusoidal modulation

$$f\left(t\right) = \sin \omega t, \tag{28}$$

produces the dynamic localization condition

$$2\frac{E_F}{\omega} = n\pi, \tag{29}$$

instead of (20). This shows that dynamic localization effects are rather sensitive to the influence of initial phases, the same "$n\pi$" realization being now produced by two times smaller field magnitudes.

Another application is the superposition

$$f\left(t\right) = \cos\left(\omega t\right) + \Delta \cos\left(2\omega t\right), \tag{30}$$

in which $\Delta$ is an arbitrary parameter. Now one finds

$$\cos\left(\omega t_q\right) = \frac{1}{4\Delta}\left[\left(1+8\Delta^2\right)^{1/2} - 1\right], \tag{31}$$

so that the dynamic localization condition exhibits a rather complex form like

$$\frac{E_F}{\omega} = n\pi F(\Delta), \tag{32}$$

where

$$F(\Delta) = \frac{\Delta\sqrt{8}}{\left[1+\frac{1}{4}\left(\left(1+8\Delta^2\right)^{1/2}-1\right)\right]\left[4\Delta^2 - 1 + \left(1+8\Delta^2\right)^{1/2}\right]^{1/2}}. \tag{33}$$

One sees that (33) reproduces (20) if $\Delta \to 0$. We have to remark that $F(\Delta)$ is a monotonically decreasing function, such that $1 \geq F(\Delta) > 0$ for $0 \leq \Delta < \infty$. It is also clear that choosing $E_F/\omega > 0$, we have to consider that $sgn(n) = sgn(\Delta)$. We emphasize that such results, which are heavily accessible by using other descriptions, deserve as quickly tractable alternatives to more involved descriptions [14].

## 5. CONCLUSIONS

In this paper an alternative approach to the dynamic localization of a charged particle moving on an one dimensional lattice under the influence of a time dependent electric field has been proposed. One proceeds by generalizing the conventional field free description, which results in the implementation of a time discretization condition such as given by (12). Under such conditions (11) shows that the wavefunction becomes an actual solution by resorting to a related time grid. In other words, the present wavefunction, which works from the very beginning on a discrete space, becomes ready for applications but under an additional time discretization only. This opens the way to derive quickly tractable but useful dynamic localization conditions by eliminating the $t_q$-parameter between (12) and (27). It is clear that this latter equation can be viewed as being responsible for an effective “field-free” behavior, which sheds some light on the generalized dynamic localization conditions established in this manner. Another version of the alternative wavefunction can be done by resorting to a slightly modified version of the method of characteristics [15].

*Acknowledgments*. We are indebted to CNCSIS/Bucharest for financial support. Interesting discussions with I. Cotaescu, Gh. Zet and I. Bica are also acknowledged.

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