

MODULATIONAL INSTABILITY OF COUPLED NONLINEAR EQUATIONS. MANAKOV'S SYSTEM

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Modulational instability of two coupled nonlinear equations (Manakov's system) is studied both from a deterministic (DAMI) and a statistical (SAMI) point of view. In SAMI kinetic equations for several two-point correlation functions are written down and their linear stability is discussed. Several initial spectral distributions (delta-spectrum, Lorentzian spectrum) are considered and explicit calculations are done for equally probable spectral functions. The influence of the statistical properties of the medium on the development of MI are easily seen: the MI is possible only if the initial state is characterized by long-range correlations.

1. INTRODUCTION

The modulational instability (MI) is a general phenomenon in the theory of nonlinear waves, generated by the interplay between nonlinearity and dispersion effects. It was discovered by Bespalov and Talanov (1966) [1] for electromagnetic waves propagating in nonlinear media and by Benjamin and Feir (1967) [2] for waves in deep waters. The phenomenon consists in the instability of a plane wave solution (a Stokes solution – a plane wave with a constant amplitude, but with an amplitude dependence of the dispersion relation) against long-scale modulation. Long time evolution leads to the growth of side bands near the fundamental wave and a mutual exchange of energy. At present MI was predicted and observed in almost all field where we are dealing with the propagation of a quasi-monochromatic wave in a weakly nonlinear medium [3–7].

Much attention has been devoted to the investigation of MI in the framework of the NLS equations. Of great theoretical and experimental importance are the situations where two or more plane waves are propagating in the medium. We mention only three such examples. One of the most known is the propagation of multiple wave train in optical fibers [8–10]. The interaction of two optical modes A_1 and A_2 in the fiber is described by the following system of coupled NLS equations

$$\begin{aligned}
i \frac{\partial A_1}{\partial t} + c_1 \frac{\partial^2 A_1}{\partial x^2} + 2 \left(\alpha |A_1|^2 + \beta |A_2|^2 \right) A_1 &= 0 \\
i \frac{\partial A_2}{\partial t} + c_2 \frac{\partial^2 A_2}{\partial x^2} + 2 \left(\beta |A_1|^2 + \gamma |A_2|^2 \right) A_2 &= 0
\end{aligned} \tag{1}$$

with $c_1, c_2, \alpha, \beta, \gamma$ real parameters. Here β is the cross coupling coefficient and the signum functions c_1, c_2 depends on the sign of the group velocity dispersion (GVD) in each mode (+1 for anomalous GVD, and -1 for normal GVD). The system was proved to be completely integrable (possesses Painlevé property) for [11]

$$\begin{aligned}
c_1 = c_2 \quad \alpha = \beta = \gamma \\
c_1 = -c_2 \quad \alpha = -\beta = \gamma
\end{aligned} \tag{2}$$

The system (1) with the restriction (first eq. 2) is nothing else but the well known completely integrable model proposed by Manakov [12]. More complicated systems and their integrability situations were investigated by several authors (see [13]). Mathematically there is a systematic way of generalizing NLS to multicomponent case (also to higher order) using group theory, which preserves the integrability structure [14].

The same system (1) appears in the theory of beat-wave accelerators, where a large-amplitude Langmuir wave is generated by the beating of two collinear lasers. For a plasma sufficiently under-dense the phase speed of the Langmuir wave is closed to the speed of light, and a relativistic particle will stay in phase with the electric field long enough time to be accelerated at high energies. Any process leading to the degradation of the Langmuir wave, like MI, will be detrimental to the acceleration process [15].

The last example is the propagation of a wave-train in shallow water [16]. Several experiments have revealed that in ocean wave spectra two or more separated peaks in the frequency domain are present. The major observation was the decay of the higher frequency peak. Starting from a KdV equation and considering the weak nonlinear interaction of two separated narrow-banded waves one arrives at the following coupled system

$$\begin{aligned}
\frac{\partial A}{\partial t} - k_1^2 \frac{\partial A}{\partial x} + ik_1 \frac{\partial^2 A}{\partial x^2} - i \frac{1}{k_1^3} |A|^2 A - 2i \frac{k_1}{k_2^4} |B|^2 A &= 0 \\
\frac{\partial B}{\partial t} - k_2^2 \frac{\partial B}{\partial x} + ik_2 \frac{\partial^2 B}{\partial x^2} - i \frac{1}{k_2^3} |B|^2 B - 2i \frac{k_2}{k_1^4} |A|^2 B &= 0
\end{aligned} \tag{3}$$

where k_1, k_2 are the wave numbers of the corresponding waves.

All these considerations justifies the study of MI for systems of coupled NLS equations. We shall do this for the Manakov system (MS)

$$\begin{aligned}
i \frac{\partial A}{\partial t} + \lambda \frac{\partial^2 A}{\partial x^2} + \mu (|A|^2 + |B|^2) A &= 0 \\
i \frac{\partial B}{\partial t} + \lambda \frac{\partial^2 B}{\partial x^2} + \mu (|A|^2 + |B|^2) B &= 0
\end{aligned} \tag{4}$$

Two distinct way to study the MI are possible. The first is a deterministic approach (DAMI), while the second is a statistical one (SAMI). In the next section DAMI will be briefly reviewed. The section three will be devoted to a careful investigation of SAMI. Few concluding remarks will be done in the last section.

2. DAMI FOR MS

The deterministic approach of modulational instability of nonlinear evolution equations is well known and can be found in almost all text book of nonlinear waves (see [3–5]). The Manakov system (4) has a Stokes wave solution $A = a \exp[i(kx - \omega t)]$, $B = b \exp[i(kx - \omega t)]$ with an amplitude dependent dispersion relation $\omega = \lambda k^2 - \mu(|a|^2 + |b|^2)$. A linear stability analysis of this solution can be done assuming

$$\begin{aligned}
A &= a(1 + \epsilon A_1) e^{i(kx - \omega t)} \\
B &= b(1 + \epsilon B_1) e^{i(kx - \omega t)}
\end{aligned} \tag{5}$$

where the amplitudes $A_1(x, t)$, $B_1(x, t)$ are satisfying the following coupled system of linear equations

$$\begin{aligned}
i \frac{\partial A_1}{\partial t} + 2i\lambda k \frac{\partial A_1}{\partial x} + \lambda \frac{\partial^2 A_1}{\partial x^2} + \mu \left[|a|^2 (A_1 + A_1^*) + |b|^2 (B_1 + B_1^*) \right] &= 0 \\
i \frac{\partial B_1}{\partial t} + 2i\lambda k \frac{\partial B_1}{\partial x} + \lambda \frac{\partial^2 B_1}{\partial x^2} + \mu \left[|a|^2 (A_1 + A_1^*) + |b|^2 (B_1 + B_1^*) \right] &= 0
\end{aligned} \tag{6}$$

Looking for plane wave solutions

$$\begin{aligned}
A_1 &= \alpha_1 e^{i(Qx - \Omega t)} + \alpha_2^* e^{-i(Qx - \Omega^* t)} \\
B_1 &= \beta_1 e^{i(Qx - \Omega t)} + \beta_2^* e^{-i(Qx - \Omega^* t)}
\end{aligned}$$

an homogeneous algebraic linear system is found for α_1 , α_2 , β_1 , β_2 . The compatibility conditions is a 4×4 determinant. Denoting $X = (\Omega - 2\lambda k Q)^2 (\lambda Q)^2$ the following equation for X is obtained

$$X^2 + 2 \left[\frac{\mu}{\lambda} (|a|^2 + |b|^2) - Q^2 \right] X - Q^2 \left[2 \frac{\mu}{\lambda} (|a|^2 + |b|^2) - Q^2 \right] = 0. \tag{7}$$

The only acceptable solution of this equation writes

$$X = - \left[2 \frac{\mu}{\lambda} (|a|^2 + |b|^2) - Q^2 \right]$$

which gives

$$\begin{aligned} \Omega_r &= 2\lambda k Q \\ |\lambda Q| \Omega_i &= \sqrt{2 \frac{\mu}{\lambda} (|a|^2 + |b|^2) - Q^2}. \end{aligned} \quad (8)$$

The modulational instability takes place in the long wave length region

$$Q < \sqrt{2 \frac{\mu}{\lambda} (|a|^2 + |b|^2) - Q^2} \quad (9)$$

if both λ and μ have the same sign, in complete agreement with the result found in the NLS case.

3. SAMI FOR MS

In the statistical approach of the modulational instability the field variables are considered as stochastic variables. The starting point is the kinetic equation for the two-point correlation function (2P-CR)

$$\rho_{\alpha\beta}(x_1, x_2) = \langle \alpha(x_1) \beta^*(x_2) \rangle \quad (10)$$

where $\alpha, \beta = A, B$ and $\langle \dots \rangle$ denotes an ensemble average by which the statistical properties of the medium are taken into account. This approach, although less used as the deterministic one, has been considered by several authors [17–19], especially in connection with the NLS equation and the study of instability phenomenon in hydrodynamics and plasma physics. Recently it was extended also to discrete systems [20, 21].

In the case of MS we have to deal with four two-point correlation functions, two diagonal ρ_{AA} and ρ_{BB} and two non-diagonal ρ_{AB} and ρ_{BA} . The non-diagonal ones are related by the relation $\rho_{BA}^*(x_1, x_2) = \rho_{AB}(x_2, x_1)$. Kinetic equations for these 2P-CR can be found as usual [17–19], and using a Gaussian approximation to decouple the four-point correlation functions we get

$$\begin{aligned} i \frac{\partial \rho_{\alpha\alpha}}{\partial t} + \lambda \left(\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} \right) \rho_{\alpha\alpha} + \mu \left[2 \left(\bar{\alpha}^2(x_1) - \bar{\alpha}^2(x_2) \right) + \bar{\beta}^2(x_1) - \bar{\beta}^2(x_2) \right] \rho_{\alpha\alpha} + \\ + \mu \left[\bar{\alpha}\bar{\beta}(x_1) \rho_{\beta\alpha} - \bar{\beta}\bar{\alpha}(x_2) \rho_{\alpha\beta} \right] = 0 \end{aligned} \quad (11)$$

for the diagonal 2P-CR and

$$i \frac{\partial \rho_{\alpha\beta}}{\partial t} + \lambda \left(\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} \right) \rho_{\alpha\beta} + \mu \left[2\bar{\alpha}^2(x_1) + \bar{\beta}^2(x_1) - \bar{\alpha}^2(x_2) - 2\bar{\beta}^2(x_2) \right] \rho_{\alpha\beta} + \mu \left[\bar{\alpha}\bar{\beta}(x_1) \rho_{\beta\beta} - \bar{\alpha}\bar{\beta}(x_2) \rho_{\alpha\alpha} \right] = 0 \quad (12)$$

for the non-diagonal components. Here $\bar{\alpha}^2(x) = \langle \alpha(x)\alpha^*(x) \rangle$ is the mean value of the squared amplitude and $\bar{\alpha}\bar{\beta}(x) = \langle \alpha(x)\beta^*(x) \rangle$ is the mean value of the crossed product of amplitudes. If $\bar{\alpha}^2(x)$ is a real quantity, $\bar{\alpha}\bar{\beta}(x)$ is a complex one and $\bar{\beta}\bar{\alpha}(x) = (\bar{\alpha}\bar{\beta}(x))^*$.

Next we shall use a Wigner-Moyal transform [22] by introducing the relative coordinate $x = x_1 - x_2$ and the center of mass coordinate $X = \frac{1}{2}(x_1 + x_2)$ and making a Fourier transform with respect to the relative coordinate

$$F_{\alpha\beta}(k, X, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ikx} \rho_{\alpha\beta}(x, X, t) dx. \quad (13)$$

From their definition it is easily shown that $F_{\beta\alpha}$ is the complex conjugate of $F_{\alpha\beta}$. Expanding all the mean values $\bar{\alpha}\bar{\beta}(x_1)$ and $\bar{\alpha}\bar{\beta}(x_2)$ in Taylor series around the point X , and using the trick

$$x^j e^{-ikx} = i^j \frac{\partial^j}{\partial k^j} e^{-ikx}$$

we find the following equations for $F_{\alpha\alpha}$ and $F_{\alpha\beta}$:

$$i \frac{\partial}{\partial t} F_{\alpha\alpha}(k, X, t) + 2ik\lambda \frac{\partial F_{\alpha\alpha}}{\partial X} + 2i\mu \sum_{j=0}^{\infty} \frac{(-)^j}{2^{2j+1} (2j+1)!} \frac{\partial^{2j+1}}{\partial X^{2j+1}} \left[2\bar{\alpha}^2(X) + \bar{\beta}^2(X) \right] \frac{\partial^{2j+1} F_{\alpha\alpha}}{\partial k^{2j+1}} + \mu \sum_{j=0}^{\infty} \frac{(i)^j}{2^j j!} \left[\frac{\partial^j}{\partial X^j} \bar{\alpha}\bar{\beta}(X) \frac{\partial^j}{\partial k^j} F_{\beta\alpha} - (-)^j \frac{\partial^j}{\partial X^j} \bar{\beta}\bar{\alpha}(X) \frac{\partial^j}{\partial k^j} F_{\alpha\beta} \right] = 0 \quad (14)$$

and

$$i \frac{\partial}{\partial t} F_{\alpha\beta}(k, X, t) + 2ik\lambda \frac{\partial F_{\alpha\beta}}{\partial X} + 2i\mu \sum_{j=0}^{\infty} \frac{(i)^j}{2^j j!} \frac{\partial^j}{\partial X^j} \left[2\bar{\alpha}^2 + \bar{\beta}^2 - (-)^j (\bar{\alpha}^2 + 2\bar{\beta}^2) \right] \frac{\partial^j F_{\alpha\beta}}{\partial k^j} + \mu \sum_{j=0}^{\infty} \frac{(i)^j}{2^j j!} \frac{\partial^j}{\partial X^j} \bar{\alpha}\bar{\beta}(X) \left[\frac{\partial^j}{\partial k^j} F_{\beta\beta} - (-)^j \frac{\partial^j}{\partial k^j} F_{\alpha\alpha} \right] = 0 \quad (15)$$

A linear stability analysis of these equation will be done. We assume

$$\begin{aligned} F_{\alpha\beta}(k, X, t) &= f_{\alpha\beta}(k) + \epsilon \mathcal{F}_{\alpha\beta}(k, X, t) \\ \overline{\alpha\beta}(X, t) &= (\overline{\alpha\beta})_0 + \epsilon (\overline{\alpha\beta})_1(X, t) \\ (\overline{\alpha\beta})_0 &= \int_{-\infty}^{+\infty} f_{\alpha\beta}(k) dk; \quad (\overline{\alpha\beta})_1 = \int_{-\infty}^{+\infty} \mathcal{F}_{\alpha\beta}(k, X, t) dk. \end{aligned} \quad (16)$$

As before $f_{\beta\alpha} = (f_{\alpha\beta})^*$, $\mathcal{F}_{\beta\alpha} = (\mathcal{F}_{\alpha\beta})^*$, $(\overline{\beta\alpha})_0 = ((\overline{\alpha\beta})_0)^*$ and $(\overline{\beta\alpha})_1 = ((\overline{\alpha\beta})_1)^*$. The diagonal $\mathcal{F}_{\alpha\alpha}$ and the non-diagonal $\mathcal{F}_{\alpha\beta}$ are satisfying now linear equations, namely

$$\begin{aligned} & i \frac{\partial}{\partial t} \mathcal{F}_{\alpha\alpha}(k, X, t) + 2ik\lambda \frac{\partial \mathcal{F}_{\alpha\alpha}}{\partial X} + \\ & + 2i\mu \sum_{j=0}^{\infty} \frac{(-)^j}{2^{2j+1}(2j+1)!} \frac{\partial^{2j+1}}{\partial X^{2j+1}} [2(\overline{\alpha^2})_1 + (\overline{\beta^2})_1] \frac{\partial^{2j+1} f_{\alpha\alpha}}{\partial k^{2j+1}} + \\ & + \mu \sum_{j=0}^{\infty} \frac{(i)^j}{2^j j!} \left[\frac{\partial^j}{\partial X^j} (\overline{\alpha\beta})_1 \frac{\partial^j}{\partial k^j} f_{\beta\alpha} - (-)^j \frac{\partial^j}{\partial X^j} (\overline{\beta\alpha})_1 \frac{\partial^j}{\partial k^j} f_{\alpha\beta} \right] = 0 \end{aligned} \quad (17)$$

and

$$\begin{aligned} & i \frac{\partial}{\partial t} \mathcal{F}_{\alpha\beta}(k, X, t) + 2ik\lambda \frac{\partial \mathcal{F}_{\alpha\beta}}{\partial X} + \\ & + \mu \sum_{j=0}^{\infty} \frac{(i)^j}{2^j j!} \frac{\partial^j}{\partial X^j} [2(\overline{\alpha^2})_1 + (\overline{\beta^2})_1 - (-)^j ((\overline{\alpha^2})_1 + 2(\overline{\beta^2})_1)] \frac{\partial^j f_{\alpha\beta}}{\partial k^j} + \\ & + \mu \sum_{j=0}^{\infty} \frac{(i)^j}{2^j j!} \frac{\partial^j}{\partial X^j} (\overline{\alpha\beta})_1 \left[\frac{\partial^j}{\partial k^j} f_{\beta\beta} - (-)^j \frac{\partial^j}{\partial k^j} f_{\alpha\alpha} \right] = 0 \end{aligned} \quad (18)$$

Let us consider plane wave solutions of these equations

$$\mathcal{F}_{\alpha\beta}(k, X, t) = g_{\alpha\beta}(k) e^{i(QX - \Omega t)} \quad (19)$$

where $g_{\alpha\beta}$ are depending now only on k . It is obvious that $g_{\alpha\alpha}$ are real and $g_{\alpha\beta}$ and $g_{\beta\alpha}$ complex conjugated. Using (16) we can write

$$\begin{aligned} (\overline{\alpha\beta})_1(X, t) &= G_{\alpha\beta} e^{i(QX - \Omega t)} \\ G_{\alpha\beta} &= \int_{-\infty}^{+\infty} g_{\alpha\beta}(k) dk. \end{aligned} \quad (20)$$

Because

$$\sum_{j=0}^{\infty} \frac{1}{j!} \left[\frac{1}{(-)^j} \right] \left(\frac{Q}{2} \right)^j \frac{\partial^j}{\partial k^j} f_{\alpha\beta}(k) = f_{\alpha\beta} \left(k \pm \frac{Q}{2} \right)$$

by introducing (19) into (17) we get

$$\begin{aligned} (\Omega - 2\lambda k Q) g_{\alpha\alpha} - \mu(2G_{\alpha\alpha} + G_{\beta\beta}) \left[f_{\alpha\alpha} \left(k + \frac{Q}{2} \right) - f_{\alpha\alpha} \left(k - \frac{Q}{2} \right) \right] + \\ + \mu G_{\alpha\beta} f_{\beta\alpha} \left(k - \frac{Q}{2} \right) - \mu f_{\alpha\beta} \left(k + \frac{Q}{2} \right) G_{\beta\alpha} = 0 \end{aligned}$$

Dividing by $(\Omega - 2\lambda k Q)$ and integrating over k we obtain

$$\begin{aligned} G_{\alpha\alpha} - \mu(2G_{\alpha\alpha} + G_{\beta\beta}) \int_{-\infty}^{+\infty} \frac{f_{\alpha\alpha} \left(k + \frac{Q}{2} \right) - f_{\alpha\alpha} \left(k - \frac{Q}{2} \right)}{\Omega - 2\lambda k Q} dk + \\ + \mu G_{\alpha\beta} \int_{-\infty}^{\infty} \frac{f_{\beta\alpha} \left(k - \frac{Q}{2} \right)}{\Omega - 2k\lambda Q} dk - \mu G_{\beta\alpha} \int_{-\infty}^{\infty} \frac{f_{\alpha\beta} \left(k + \frac{Q}{2} \right)}{\Omega - 2k\lambda Q} dk = 0. \end{aligned} \quad (21)$$

In a similar way the following relation is found for the non-diagonal components $G_{\alpha\beta}$

$$\begin{aligned} G_{\alpha\beta} \left(1 - \mu \int_{-\infty}^{+\infty} \frac{f_{\alpha\alpha} \left(k + \frac{Q}{2} \right)}{\Omega - 2\lambda k Q} dk + \mu \int_{-\infty}^{+\infty} \frac{f_{\beta\beta} \left(k - \frac{Q}{2} \right)}{\Omega - 2\lambda k Q} dk \right) + \\ + \mu G_{\alpha\alpha} \left(2 \int_{-\infty}^{+\infty} \frac{f_{\alpha\beta} \left(k - \frac{Q}{2} \right)}{\Omega - 2\lambda k Q} dk - \int_{-\infty}^{+\infty} \frac{f_{\alpha\beta} \left(k + \frac{Q}{2} \right)}{\Omega - 2\lambda k Q} dk \right) - \\ - \mu G_{\beta\beta} \left(2 \int_{-\infty}^{+\infty} \frac{f_{\alpha\beta} \left(k + \frac{Q}{2} \right)}{\Omega - 2\lambda k Q} dk - \int_{-\infty}^{+\infty} \frac{f_{\alpha\beta} \left(k - \frac{Q}{2} \right)}{\Omega - 2\lambda k Q} dk \right) = 0. \end{aligned} \quad (22)$$

From (22) $G_{\alpha\beta}$ and $G_{\beta\alpha}$ are easily obtained and introduced into (21) lead to an homogeneous system of two linear coupled equations for $G_{\alpha\alpha}$ and $G_{\beta\beta}$. The integral stability equations we are looking for is given by the compatibility condition of this system.

Explicit calculations will be done for specific initial spectrum. The first example is a δ -spectrum

$$f_{\alpha\beta} = (\overline{\alpha\beta})_0 \delta(k). \quad (23)$$

We introduce the notations $(\overline{AA})_0 = a^2$, $(\overline{BB})_0 = b^2$, $(\overline{AB})_0 = abc^{i\Phi}$ and $(\overline{BA})_0 = abc^{-i\Phi}$ with a and b real quantities. Considering Ω purely imaginary

($\Omega = i\Omega_i$) and denoting $\omega_i = \frac{\Omega_i}{2\lambda Q}$ the integrals in (21) and (22) are straightforward. After several manipulations the compatibility conditions writes

$$MP - NP = 0 \quad (24)$$

where

$$\begin{aligned} M &= \omega_i^2 + \frac{Q^2}{4} - 2a^2 - a^2b^2 \left(\frac{\mu}{D} + \frac{\mu^*}{D^*} \right) \\ N &= a^2 + a^2b^2 \left(\frac{\nu}{D} + \frac{\nu^*}{D^*} \right) \\ P &= \omega_i^2 + \frac{Q^2}{4} - 2b^2 - a^2b^2 \left(\frac{\mu^*}{D} + \frac{\mu}{D^*} \right) \\ Q &= b^2 + a^2b^2 \left(\frac{\nu^*}{D} + \frac{\nu}{D^*} \right) \\ D &= \omega_i^2 + \frac{Q^2}{4} - \frac{a^2 + b^2}{2} + i \frac{\omega_i}{Q} (a^2 - b^2) \\ \mu &= \frac{3}{4} - \frac{\omega_i^2}{Q^2} + 2i \frac{\omega_i}{Q} \\ \nu &= \frac{3}{4} + \frac{\omega_i^2}{Q^2} + i \frac{\omega_i}{Q}. \end{aligned} \quad (25)$$

A very simple result is obtained if $a = b$. Straightforward calculations give

$$\omega_i = \sqrt{4a^2 - \frac{Q^2}{4}} \quad (26)$$

and the instability is possible in the long wave-length region if $Q < 4a$.

A more realistic situation is a Lorentzian spectrum

$$f_{\alpha\beta}(k) = \frac{(\overline{\alpha\beta})_0}{\pi} \frac{p}{k^2 + p^2}. \quad (27)$$

It is easily seen that the result in this case is obtained from the result of δ -spectrum changing ω_i in $(\omega_i + p)$. In the case $a = b$ we find

$$\omega_i = \sqrt{4a^2 - \frac{Q^2}{4}} - p \quad (28)$$

which shows that for p greater than the square root $\sqrt{4a^2 - \frac{Q^2}{4}}$ the instability do not exists. Therefore the MI is possible only if the initial state is characterized by some long-range correlations, conclusion which seems to have an universal validity.

4. CONCLUSIONS

The MI is the first step in the generation of soliton like excitations in physical systems. Therefore the study of the conditions in which this phenomenon takes place is of special importance. The statistical approach takes into account the statistical properties of the medium, giving restriction for the development of MI. The general conclusion obtained from the analysis of NLS type equations [16–21] is that MI appears only if a certain long-range correlation exists in the initial state. The same conclusion is obtained here from the study of Manakov's system and it seems to be of general validity.

In the NLS case the MI is also restricted to the long-wave-length region. But recent results on derivative NLS equation [23] have shown that this conclusion is not generally valid. Contrary to the NLS case for derivative NLS the instability region is strongly dependent on the wave vector of carrying wave. A DAMI analysis for derivative NLS equation

$$i \frac{\partial \Psi}{\partial t} + \alpha \frac{\partial^2 \Psi}{\partial x^2} + i\beta |\Psi|^2 \frac{\partial \Psi}{\partial x} = 0$$

gives [23]

$$\Omega = (2\alpha k + \beta |a|^2) Q + i\alpha Q \sqrt{2 \frac{\beta}{\alpha} |a|^2 k - Q^2}$$

and the instability is possible if $Q^2 < 2 \frac{\beta}{\alpha} |a|^2 k$ depending on k , contrary to the NLS case when $Q^2 < 2 \frac{\beta}{\alpha} |a|^2$. So the type of nonlinearity can have strong influence on the MI phenomenon.

In the present paper the MI was carefully studied for Manakov's system. Having the same type of nonlinearity it is expected that all the conclusions obtained for NLS equation remain valid also in this case. The problem is complicated by the fact that we have to deal with four correlation functions and a result for the most general case is not yet obtained. But for a special case of "complete democracy" (equal amplitudes in the initial state) the results obtained here are in complete agreement with the NLS case.

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