TELEBROADCASTING OF ENTANGLED TWO-SPIN-1/2 STATES

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Received December 10, 2004

A quantum telebroadcasting process combining the many-to-many teleportation and asymmetric broadcasting of entanglement from one pair of observers to two spatially separated pairs of observers is presented. By applying the Peres-Horodecki criterion we analyze the inseparability of the final states and show that this depends on the parameter, which characterizes the quantum channel used in the process. The final inseparable states represent the output states generated in the broadcasting of entanglement *via* local cloning.

1. INTRODUCTION

Quantum teleportation is the basic ingredient for many communication processes. It performs the transmission and reconstruction of an unknown quantum state over arbitrary distances with the help of entangled states. In the standard teleportation scheme introduced by Bennett *et al.* [1], the state is transfered from one sender, Alice, to one receiver, Bob.

In this paper we review two generalizations of quantum teleportation: one-to-many and many-to-many teleportation, where the information of a quantum system is distributed from one sender to M receivers, and from N senders to M receivers, respectively (Section 2). Then, in Section 3 we present the Peres-Horodecki criterion of separability of mixed two-spin-1/2 particles. A summary of broadcasting of entanglement using local optimal universal asymmetric cloners is given in Section 4.1. In Section 4.2 we present the telebroadcasting of two-spin-1/2 entangled states to two distant pairs of observers.

2. QUANTUM TELEPORTATION

2.1. ONE-TO-ONE AND ONE-TO-MANY TELEPORTATION

Let us start by reviewing the original teleportation protocol and its generalization, one-to-many teleportation. In the standard teleportation scheme

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an unknown qudit (a state of a d-level system) is faithfully transmitted from one observer, Alice, to another observer, Bob, while the initial Alice's state is destroyed.

Let the initial unknown state we wish to teleport be $|\psi\rangle = \sum_{k=0}^{d-1} \alpha_k |k\rangle_A$, where $\sum_{k=0}^{d-1} |\alpha_k|^2 = 1$, and $\{|k\rangle\}$ is the computational basis. The quantum channel required in this process is a maximally entangled state shared by Alice and Bob

$$|\xi\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |j\rangle_A |j\rangle_B . \tag{2.1}$$

The state of the whole system of the three particles is:

$$|\psi\rangle|\xi\rangle = \frac{1}{d} \sum_{m=0}^{d-1} \sum_{n=0}^{d-1} |\Phi_{m,n}\rangle \sum_{k=0}^{d-1} \exp\left(-\frac{2\pi ikn}{d}\right) \alpha_k |\overline{k+m}\rangle, \tag{2.2}$$

where $\overline{k+m} = k+m$ modulo d, and

$$|\Phi_{m,n}\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \exp\left(\frac{2\pi ikn}{d}\right) |k\rangle |\overline{k+m}\rangle, \tag{2.3}$$

is the generalized Bell basis [1, 2]. Alice performs a Bell-type measurement on her particles and sends the result to Bob. If the outcome of Alice's measurement is $|\Phi_{m,n}\rangle$, then Bob has to apply the unitary operator [1]

$$V_{m;n} = \sum_{j=0}^{d-1} \exp\left(\frac{2\pi ijn}{d}\right) |j\rangle \langle \overline{j+m}| \tag{2.4}$$

on his particle in order to retrieve the initial state. Having performed the Bell-type measurement, Alice destroyed the information contained in the initial unknown state, as it must be conformable to the no-cloning theorem. The protocol introduced by Bennett *et al.* [1] is called one-to-one teleportation since the information is transmitted from one sender to one receiver.

We now briefly present the one-to-many teleportation protocol proposed by Murao *et al.* [3], where the information is distributed from one sender, Alice, to M distant receivers, B_1, B_2, $\ldots$, B_M, using multiparty entanglement. The information of a d-level system encoded in an N-particle state is:

$$|\psi\rangle = \sum_{k=0}^{d-1} \alpha_k |\psi_k\rangle_A , \tag{2.5}$$

with $\sum_{k=0}^{d-1} |\alpha_k|^2 = 1$, and $\{|\psi_k\rangle_A\}$ represents a basis in the d-dimensional space. The quantum channel is a maximally entangled state of the Alice's N particles and receivers' M particles:

$$|\xi\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |\pi_j\rangle_A |\phi_j\rangle_{B_1 B_2 \ldots B_M}, \tag{2.6}$$

where $\{|\pi_j\rangle\}$ and $\{|\phi_j\rangle\}$ are bases in the d-level spaces of Alice's and receivers' particles, respectively. Alice performs a joint measurement on her particles in the generalized Bell basis. Depending on the result communicated by Alice, the receivers apply a local "recovery unitary operation" (RUO) [3]. If the result of Alice's measurement is $|\Phi_{m;n}\rangle$, then the receivers perform the unitary operation that satisfies the condition:

$$V_{m;n} = \mathcal{V}_{B_1} \otimes \mathcal{V}_{B_2} \ldots \otimes \mathcal{V}_{B_M} = \sum_{j=0}^{d-1} \exp\left(\frac{2\pi i j n}{d}\right) |\phi_j\rangle\langle\phi_{\overline{j+m}}|. \tag{2.7}$$

Therefore, the information of the initial unknown state of Eq. (2.5) has been distributed to several distant parties:

$$|\phi\rangle = \sum_{j=0}^{d-1} \alpha_j |\phi_j\rangle_{B_1 B_2 \ldots B_M}. \tag{2.8}$$

2.2. MANY-TO-MANY TELEPORTATION

Now we present the many-to-many teleportation of a d-level system proposed by us in Ref. [4]. In this protocol the information of a d-level system initially shared by N distant observers is transmitted to M distant receivers (with $M > N$). The initial entangled state of the observers $A_1, A_2, \ldots, A_N$ is given by

$$|\psi\rangle = \sum_{k=0}^{d-1} \alpha_k |\psi_k\rangle_{A_1} |\psi_k\rangle_{A_2} \ldots |\psi_k\rangle_{A_N}, \tag{2.9}$$

with $\sum_{k=0}^{d-1} |\alpha_k|^2 = 1$, and $\{|\psi_k\rangle_{A_j}\}$ represents a basis in the d-dimensional space of the jth sender.

We define the quantum channel as a maximally entangled $(N + M)$-particle state shared between senders and receivers:

$$\left|\xi\right\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}\left|\pi_j\right\rangle_{A'_1}\left|\pi_j\right\rangle_{A'_2}\ldots\left|\pi_j\right\rangle_{A'_N}\left|\phi_j\right\rangle_{B_1B_2\ldots B_M}, \tag{2.10}$$

where we have denoted by '*B*' the particles that belong to the receivers. The states $\left\{\left|\pi_j\right\rangle_{A'_1}\right\}$ represent a *d*-dimensional basis for the *i*th sender. The joint state of the initial system and the channel is

$$\begin{aligned}\left|\psi\right\rangle\left|\xi\right\rangle &= \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}\alpha_k\sum_{j=0}^{d-1}\left|\psi_k\right\rangle_{A_1}\left|\pi_j\right\rangle_{A'_1}\left|\psi_k\right\rangle_{A_2}\left|\pi_j\right\rangle_{A'_2}\ldots\left|\psi_k\right\rangle_{A_N}\left|\pi_j\right\rangle_{A'_N}\\ &\otimes\left|\phi_j\right\rangle_{B_1\ldots B_M}\\ &= \frac{1}{d^{\frac{N+1}{2}}}\sum_m\sum_{n_1,n_2,\ldots,n_N}\left|\Phi_{m,n_1}\right\rangle\left|\Phi_{m,n_2}\right\rangle\ldots\left|\Phi_{m,n_N}\right\rangle\\ &\times\sum_k\exp\left[-\frac{2\pi ik}{d}(n_1+n_2+\ldots+n_N)\right]\alpha_k\left|\phi_{\overline{k+m}}\right\rangle.\end{aligned} \tag{2.11}$$

The protocol for many-to-many teleportation is the following:

a) Each sender performs a measurement of his particles in the generalized Bell basis.

b) The senders communicate the result of the measurement to the *M* receivers.

c) Let us analyze the case when the outcome of the senders' Bell measurement is:

$$\left|\Phi_{m,n_1}\right\rangle\left|\Phi_{m,n_2}\right\rangle\ldots\left|\Phi_{m,n_N}\right\rangle. \tag{2.12}$$

Then, the receivers have to apply a local recovery unitary operation that fulfills:

$$V_{m;n_1,n_2,\ldots,n_N}\left|\phi_k\right\rangle = \exp\left[\frac{2\pi ik}{d}(n_1+n_2+\ldots+n_N)\right]\left|\phi_{\overline{k-m}}\right\rangle. \tag{2.13}$$

Therefore, the many-to-many teleportation distributes the information of the initial *N*-particle state (2.9) into the *M*-particle state:

$$\left|\psi\right\rangle = \sum_{j=0}^{d-1}\alpha_j\left|\psi_j\right\rangle_{A_1}\left|\psi_j\right\rangle_{A_2}\ldots\left|\psi_j\right\rangle_{A_N}\rightarrow\left|\phi\right\rangle = \sum_{j=0}^{d-1}\alpha_j\left|\phi_j\right\rangle_{B_1B_2\ldots B_M}. \tag{2.14}$$

3. THE PERES-HORODECKI CRITERION OF SEPARABILITY

In this section we present the necessary and sufficient condition for the separability of mixed two-spin-1/2 particles. Let us recall the definition of the separability of a bipartite system: A state is separable if the density operator describing this state can be written as a convex combination of product states [5]:

$$\rho = \sum_i p_i \rho_i^{(1)} \otimes \rho_i^{(2)}, \tag{3.15}$$

where $\rho_i^{(1)}$ and $\rho_i^{(2)}$ are density operators of the first subsystem, and second subsystem, respectively.

Theorem (Peres-Horodecki). A two-spin-1/2 state is separable if and only if the partial transposition of the density operator is a nonnegative one [6, 7].

While the necessary condition found by Peres is valid for arbitrary bipartite mixed states, the sufficient one is true only for 2 × 2 and 2 × 3 systems.

4. TELEBROADCASTING OF A BIPARTITE TWO-LEVEL ENTANGLED STATE

4.1. PRELIMINARIES

The no-cloning theorem forbids the existence of a unitary operation that can produce two perfect copies of an arbitrary quantum state [8]. Therefore some approximate methods for cloning were proposed, where the fidelity between the final identical states and the initial one is less than unity [9, 10, 2]. In the case of asymmetric cloning (when the two final clones are not identical), it is interesting when the universal cloning machine is optimal, that means a machine that creates the second clone with maximal fidelity for a given fidelity of the first one [2, 11]. Cerf has found the expression of the optimal universal asymmetric cloning machine of d-level states using a reference state [2]. We have also obtained an equivalent expression of this cloning machine, by eliminating the reference state [4]:

$$U|j\rangle|00\rangle = \frac{1}{\sqrt{1+(d-1)(p^2+q^2)}}\Big(|j\rangle|j\rangle|j\rangle + \\ + p\sum_{r=1}^{d-1}|j\rangle|\overline{j+r}\rangle|\overline{j+r}\rangle + q\sum_{r=1}^{d-1}|\overline{j+r}\rangle|j\rangle|\overline{j+r}\rangle\Big), \tag{4.16}$$

where $p + q = 1$.

An interesting application of quantum cloning is broadcasting of entanglement proposed by Bužek *et al.* [12]. In this process, the entanglement originally shared by two observers is broadcast into two identical entangled states by using local $1 \to 2$ optimal universal symmetric cloning machine. We have investigated the broadcasting of entanglement using the optimal universal *asymmetric* cloning machine by employing the formula (4.16) for $d = 2$:

$$U(p)|0\rangle|00\rangle = \frac{1}{\sqrt{1+p^2+q^2}}\left(|000\rangle + p|011\rangle + q|101\rangle\right)$$
$$U(p)|1\rangle|00\rangle = \frac{1}{\sqrt{1+p^2+q^2}}\left(|111\rangle + p|100\rangle + q|010\rangle\right), \tag{4.17}$$

with $p + q = 1$, where the first two qubits represent the clones and the last one is the ancilla [4]. The initial entanglement shared by two observers, Alice and Bob is:

$$|\psi\rangle_{12} = \alpha|00\rangle + \beta|11\rangle. \tag{4.18}$$

The state of the total system, consisting of the two particles '1' and '2', and another four particles: the blank states '3' and '4', the ancillas '5', '6' is given by $|\Pi\rangle = U(p)\otimes U(p)|\psi\rangle_{12}|00\rangle_{35}|00\rangle_{46}$, where the particles denoted by odd number belong to Alice, while the even particles belong to Bob. By using the Peres-Horodecki criterion we have evaluated in Ref. [4] the inseparability of the two final states ρ_{14} and ρ_{23}.

4.2. TELEBROADCASTING OF TWO-SPIN-1/2 ENTANGLED STATES

Consider that two spatially separated observers, A_1 and A_2, hold an entangled state and they wish to teleport two copies of this state to two pairs of observers also located at different places, B_1–B_4, and B_2–B_3, respectively.

Suppose that A_1 and A_2 share an arbitrary two-spin-1/2 entangled state

$$|\psi\rangle_{A_1A_2} = \alpha|00\rangle + \beta|11\rangle. \tag{4.19}$$

Here we use the notation spin up $|\uparrow\rangle = |0\rangle$ and spin down $|\downarrow\rangle = |1\rangle$. We propose a new scheme called telebroadcasting of entanglement, which simultaneously copy and transfer the information of the initial entangled state. This protocol combines the many-to-many teleportation and asymmetric broadcasting of entanglement.

We define two six-particle states:

$$|\phi_0\rangle := \frac{1}{1+p^2+q^2}\big(|000000\rangle + p|000101\rangle + q|010001\rangle + \\ +p|001010\rangle + p^2|001111\rangle + pq|011011\rangle + \\ +q|100010\rangle + pq|100111\rangle + q^2|110011\rangle\big); \tag{4.20}$$

$$|\phi_1\rangle := \frac{1}{1+p^2+q^2}\big(|111111\rangle + p|111010\rangle + q|101110\rangle + \\ +p|110101\rangle + p^2|110000\rangle + pq|100100\rangle + \\ +q|011101\rangle + pq|011000\rangle + q^2|001100\rangle\big). \tag{4.21}$$

We choose the multiparticle quantum channel required in the many-to-many protocol as:

$$|\xi\rangle = \frac{1}{\sqrt{2}}|00\rangle_{A_1'A_2'}|\phi_0\rangle_{B_1B_2B_3B_4B_5B_6} + \frac{1}{\sqrt{2}}|11\rangle_{A_1'A_2'}|\phi_1\rangle_{B_1B_2B_3B_4B_5B_6}, \tag{4.22}$$

where B_5, B_6 are two distant observers.

The total state is

$$\begin{aligned}|\psi\rangle|\xi\rangle = \frac{1}{2\sqrt{2}}\Big[&|\Phi^+\rangle|\Phi^+\rangle\big(\alpha|\Phi_0\rangle+\beta|\Phi_1\rangle\big)+\\ &+|\Phi^+\rangle|\Phi^-\rangle\big(\alpha|\phi_0\rangle+\beta|\phi_1\rangle\big)+\\ &+|\Phi^-\rangle|\Phi^+\rangle\big(\alpha|\phi_0\rangle-\beta|\phi_1\rangle\big)+|\Phi^-\rangle|\Phi^-\rangle\big(\alpha|\phi_0\rangle+\beta|\phi_1\rangle\big)+\\ &+|\Psi^+\rangle|\Psi^-\rangle\big(\alpha|\phi_1\rangle+\beta|\phi_0\rangle\big)+|\Psi^+\rangle|\Psi^-\rangle\big(\alpha|\phi_1\rangle-\beta|\phi_0\rangle\big)+\\ &+|\Psi^-\rangle|\Psi^+\rangle\big(\alpha|\phi_1\rangle-\beta|\phi_0\rangle\big)+\\ &+|\Psi^-\rangle|\Psi^-\rangle\big(\alpha|\phi_1\rangle+\beta|\phi_0\rangle\big)\Big].\end{aligned} \tag{4.23}$$

The many-to-many protocol consists of three steps, as was shown in Sec. 2.2:

a) A_1 and A_2 perform a measurement of the particles available in the Bell basis.

b) A_1 and A_2 communicate the outcomes to the six receivers, B_1, B_2, B_3, B_4, B_5, B_6.

c) The receivers apply local unitary operations depending on the outcomes of the senders' measurements. In Table 1 we have shown the local recovery unitary operations that have to be performed for each outcome of the Bell measurements, which satisfies Eq. (2.13).

Hence, the information of the inial state (4.19) is encoded in the final state shared by the six receivers:

$$|\mu\rangle = \alpha|\phi_0\rangle_{B_1B_2B_3B_4B_5B_6} + \beta|\phi_1\rangle_{B_1B_2B_3B_4B_5B_6}. \tag{4.24}$$

We say that the input state $|\psi\rangle_{12}$ has been telebroadcast if the following two necessary conditions are satisfied [4]:

(i) the local reduced density operators $\rho_{B_1B_3}$ and $\rho_{B_2B_4}$ are separable,

(ii) the nonlocal states $\rho_{B_1B_4}$ and $\rho_{B_2B_3}$ are inseparable.

Applying the Peres-Horodecki theorem presented in Section 3, we get the condition for the separability of the local states:

Table 1

The local recovery unitary operations that have to be applied by the receivers, which depend on the outcomes of the senders' measurements

The outcome	The local recovery unitary operation
$\|\Phi^+\rangle\|\Phi^+\rangle$	$I\otimes I\otimes I\otimes I\otimes I\otimes I$
$\|\Phi^+\rangle\|\Phi^-\rangle$	$\sigma_z\otimes I\otimes\sigma_z\otimes I\otimes\sigma_z\otimes I$
$\|\Phi^-\rangle\|\Phi^+\rangle$	$\sigma_z\otimes I\otimes\sigma_z\otimes I\otimes\sigma_z\otimes I$
$\|\Phi^-\rangle\|\Phi^-\rangle$	$I\otimes I\otimes I\otimes I\otimes I\otimes I$
$\|\Psi^+\rangle\|\Psi^+\rangle$	$\sigma_x\otimes\sigma_x\otimes\sigma_x\otimes\sigma_x\otimes\sigma_x\otimes\sigma_x$
$\|\Psi^+\rangle\|\Psi^-\rangle$	$\sigma_y\otimes\sigma_x\otimes\sigma_y\otimes\sigma_x\otimes\sigma_y\otimes\sigma_x$
$\|\Psi^-\rangle\|\Psi^+\rangle$	$\sigma_y\otimes\sigma_x\otimes\sigma_y\otimes\sigma_x\otimes\sigma_y\otimes\sigma_x$
$\|\Psi^-\rangle\|\Psi^-\rangle$	$\sigma_x\otimes\sigma_x\otimes\sigma_x\otimes\sigma_x\otimes\sigma_x\otimes\sigma_x$

$$\alpha^2\beta^2-p^2q^2\geq 0 \tag{4.25}$$

or equivalent

$$\frac{1}{2}\left[1-\sqrt{1-4p^2(1-p)^2}\right]\leq\alpha^2\leq\frac{1}{2}\left[1+\sqrt{1-4p^2(1-p)^2}\right]. \tag{4.26}$$

The density operators of the nonlocal states are:

$$\begin{aligned}\rho_{B_1B_4}=&\frac{1}{(1+p^2+q^2)^2}\{[p^2q^2+\alpha^2(1+p^2+q^2)]|00\rangle\langle 00|+[p^2q^2+\\&+\beta^2(1+p^2+q^2)]|11\rangle\langle 11|+4pq\alpha\beta(|00\rangle\langle 11|+|11\rangle\langle 00|)+\\&+(\beta^2q^4+\beta^2q^2+\alpha^2p^4+\alpha^2p^2)|01\rangle\langle 10|+\\&+(\beta^2p^4+\beta^2p^2+\alpha^2q^4+\alpha^2q^2)|10\rangle\langle 10|\},\end{aligned} \tag{4.27}$$

$$\begin{aligned}\rho_{B_2B_3}=&\frac{1}{(1+p^2+q^2)^2}\{[p^2q^2+\alpha^2(1+p^2+q^2)]|00\rangle\langle 00|+[p^2q^2+\\&+\beta^2(1+p^2+q^2)]|11\rangle\langle 11|+4pq\alpha\beta(|00\rangle\langle 11|+|11\rangle\langle 00|)+\\&+(\beta^2p^4+\beta^2p^2+\alpha^2q^4+\alpha^2q^2)|01\rangle\langle 01|+\\&+(\beta^2q^4+\beta^2q^2+\alpha^2p^4+\alpha^2p^2)|10\rangle\langle 10|\},\end{aligned} \tag{4.28}$$

Again we use the Peres-Horodecki criterion and find that the two nonlocal states are inseparable if

$$\begin{aligned}&(\beta^2p^4+\beta^2p^2+\alpha^2q^4+\alpha^2q^2)(\beta^2q^4+\beta^2q^2+\alpha^2p^4+\alpha^2p^2)-\\&-16\alpha^2\beta^2p^2q^2\leq 0\end{aligned} \tag{4.29}$$

or equivalent

$$\frac{1}{2}\left(1-\sqrt{1-4\lambda}\right)\le\alpha^2\le\frac{1}{2}\left(1+\sqrt{1-4\lambda}\right), \tag{4.30}$$

where

$$\lambda=\frac{p^4q^4+p^2q^4+p^4q^2+p^2q^2}{2p^4q^4+2p^4q^2+2p^2q^4-q^8-2q^6-q^4-p^8-2p^6-p^4+18p^2q^2}. \tag{4.31}$$

The requirements that $1-4\lambda$ has to be positive and the local states are separable when the nonlocal ones are inseparable lead to [4]:

$$\frac{1}{2}\left(1-\sqrt{-9+2\sqrt{21}}\right)\le p\le\frac{1}{2}\left(1+\sqrt{-9+2\sqrt{21}}\right). \tag{4.32}$$

The final states (4.27) and (4.28) obtained by the four receivers represent the output states generated in broadcasting of entanglement using local optimal universal asymmetric cloning machines described in Sec. 4.1.

Our process, telebroadcasting of entanglement, performs the teleportation of the final inseparable states obtained using local broadcasting to two pairs of distant observers.

In conclusion, we have proven how one can transmit optimal information of an entangled state to two pairs of receivers using only local operations and classical communication.

Acknowledgment. This work was supported by the Romanian CNCSIS through a grant for the University of Bucharest.

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