UNIFIED SELF-DUAL GAUGE THEORY OF GRAVITATIONAL AND ELECTROMAGNETIC FIELDS

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Received December 10, 2004

A gauge theory with $SO(1,4)\times U(1)$ as structural group and having a spherically symmetric Minkowski space-time as base manifold is constructed. $SO(1,4)$ denotes the de-Sitter (*DS*) group and $U(1)$ is the abelian group of phase transformations. The direct-product structure of the structural group imposes the introduction of the tetrads and spin connection as gauge fields (or potentials) of the gravitational field and of the potential vector describing the electromagnetic field in our unified gauge theory. The strength tensors of these gauge potentials are calculated considering a model with spherically symmetric fields. Then, the self-duality equations are obtained and their solutions are analyzed. It is shown that these equations admit the *DS* solution. In contrast, it is established that the Reissner-Nordström (*RN*) solution is not a self-dual one. However, the Yang-Mills field equations have both *DS* and *RN* as their solutions.

1. INTRODUCTION

It is well known that the interactions of different fields, such as electromagnetic field or gravitational field, can be closely related to the invariance properties of the theory used to describe them. Thus, if the Langrangian of matter fields is invariant under phase transformations with constant parameter α, then the electromagnetic field can be introduced by demanding invariance under the local transformations, obtained by replacing α with a function of space-time coordinates, $\alpha \to \alpha(x)$ [1]. This idea was extend by Yang and Mills to the case of non-abelian $SU(2)$ symmetry [2]. Studying these internal local gauge symmetries is particularly interesting when we try the generalization to local space-time symmetries.

On the other hand, it is much less known that the General Relativity (*GR*) theory can be formulated as a gauge theory. It is supposed that the theory is invariant under local Poincaré transformations. Then, it is necessary to introduce new compensating fields, describing the gravitational interactions [3].

Although the Poincaré gauge theory leads to a satisfactory classical theory of gravity, the analogy with gauge theories of internal symmetries is not perfect,

Rom. Journ. Phys., Vol. 50, Nos. 1–2, P. 63–73, Bucharest, 2005

because of the specific treatment of translations [4]. It is possible, however, to formulate the gauge theory of gravity in a way that treats the whole Poincaré group in a more unified framework. The approach is based on the de-Sitter (*DS*) group and the Lorentz and translation parts are distinguished through a mechanism of spontaneous symmetry breaking [5].

The *DS* group has the interesting property that in a special limit, when a deformation parameter λ of the group tends to zero, it reduces to the Poincaré group. The matter fields are described by an action that is invariant under the global $DS = SO(1, 4)$ symmetry and the gravity is introduced as a gauge field in the process of localization of this symmetry [6]. The parameter λ determines the cosmological constant that can not be introduced in the Poincaré gauge theory. So that the Poincaré symmetry is a good symmetry at all but cosmological scale [7].

If we look to a unified theory of gravity and electromagnetism, then we have to use the direct product $SO(1,4)\times U(1)$ group as the global symmetry of the integral action. Then, localizing this symmetry one introduces both the gravity and electromagnetism as gauge fields. In this paper we develop such an unified gauge theory and search the self-dual solutions for the gauge fields.

In Sect. 2 we give the general formulation of the gauge theory with $SO(1,2)\times U(1)$ as structural group and with a spherically symmetric Minkowski space-time as base manifold. The equations of structure for this group are written and then the strength tensors associated to the gauge potentials $A_\mu^{ab}(x)$ (describing the gravitational field) and $A_\mu(x)$ (the potential vector of the electromagnetic field) are calculated.

Section 3 is devoted to the study of a model of $SO(1,4)\times U(1)$ gauge theory with spherical symmetry. The gauge potentials $A_\mu^{ab}(x)$ and $A_\mu(x)$ are supposed to depend only of the $3D$ radius r of the Minkowski space-time. Then the components of the strength tensors are calculated both for the gravitational and electromagnetic fields. The source of the gravitational field is supposed to be a point-like mass m which has also an electrical charge Q.

The self-duality equations are presented in Sect. 4. It is shown that these equations admit the *DS* solution, but no the Reissner-Nordström (*RN*) one. The cosmological constant is introduced and its expression as a function of the deformation parameter λ is obtained.

To compare the self-dual solutions with other known results we write the Yang-Mills field Equations in Sect. 5. We prove that these equations admit both *DS* and *RN* as their solutions. In Sect. 6 we give the concluding remarks and suggest some open questions in a unified gauge theory of gravity with other fundamental interactions.

2. $SO(1,4)\times U(1)$ GAUGE THEORY

The *DS* group has the dimension equal to ten and the $U(1)$ group of phase transformations is abelian, one-dimensional. The infinitesimal generators of the *DS* group will be denoted by $M_{ab}=-M_{ba}$, $a,b=0,i$, $i=0,1,2,3$, and they include both the Lorentz generators $M_{ij}=-M_{ji}$ and the de-Sitter "translations" Π_i. The $U(1)$ group has the unity I as its unique generator. The equations of structure have the form [4, 6]:

$$\left[M_{ab},M_{cd}\right]=\eta_{bc}M_{ad}-\eta_{ac}M_{bd}-\eta_{bd}M_{ac}+\eta_{ad}M_{bc} \tag{2.1a}$$

$$\left[M_{ab},I\right]=0,\quad \left[I,I\right]=0. \tag{2.1b}$$

Here $\eta_{ab}=(1,-1,-1,-1,-1)$ is the Lorentz metric of a five-dimensional space M_5 over that the de-Sitter group is defined. If we use M_{ij} and Π_i, $i,j=0,1,2,3$, as generators of the de-Sitter group $SO(1,4)$ then the equations of structure (2.1) become:

$$\left[M_{mn},M_{\ell r}\right]=\eta_{n\ell}M_{mr}-\eta_{m\ell}M_{nr}-\eta_{nr}M_{m\ell}+\eta_{mr}M_{nl}, \tag{2.2a}$$

$$\left[M_{mn},\Pi_{\ell}\right]=\eta_{n\ell}\Pi_m-\eta_{m\ell}\Pi_n, \tag{2.2b}$$

$$\left[\Pi_m,\Pi_n\right]=-4\lambda^2\Sigma_{mn}, \tag{2.2c}$$

$$\left[\Sigma_{mn},I\right]=\left[\Pi_m,I\right]=\left[I,I\right]=0, \tag{2.2d}$$

with $m,n,l,r=0,1,2,3$.

A matter field $\phi(x)$ is always referred to a local frame L of the Minkowski space-time. In general, it is a multicomponent object which can be represented as a vector-column. The action of the global de-Sitter group, in the tangent space, transforms an L frame into another L frame and determine an appropriate transformation of the field $\phi(x)$ [4]:

$$\phi'(x')=\left(1+\frac{1}{2}\lambda^{ab}\Sigma_{ab}\right)\phi(x'). \tag{2.3}$$

The spin matrix Σ_{ab} is related to the multicomponent structure of $\phi(x)$. Equivalent, the relations (2.3) can be written as:

$$\delta\phi=\left(\varepsilon^a\Pi_a+\frac{1}{2}\lambda^{ab}M_{ab}\right)\phi(x), \tag{2.4}$$

where $\delta\phi=\phi'(x')-\phi(x)$.

We define now the gauge covariant derivative, associated to the local group of symmetry $SO(1,4)\times U(1)$:

$$\nabla_\mu\phi(x)=\left(\partial_\mu+\frac{g'}{2}A_\mu^{ab}\Sigma_{ab}+g''A_\mu I\right)\phi(x). \tag{2.5}$$

Here, $A_\mu^{ab}(x)=-A_\mu^{ba}(x)$ and $A_\mu(x)$ are the gauge potentials describing the gravitational and electromagnetic fields. The quantities g' and g'' denote the coupling constants for gravitational and respectively electromagnetic field. Then, we calculate the commutator $\left[\nabla_\mu,\nabla_\nu\right]$ in order to obtain the expressions of the strength tensors. We have:

$$\begin{aligned}\left[\nabla_\mu,\nabla_\nu\right]\phi(x)=\Big\{&\frac{g'}{2}\left[\partial_\mu A_\nu^{ab}-\partial_\nu A_\mu^{ab}+g'(A_{c\mu}^{a}A_\nu^{cb}-A_{c\nu}^{a}A_\mu^{cb})\right]\Sigma_{ab}+\\&+g''\left(\partial_\mu A_\nu-\partial_\nu A_\mu\right)\Big\}\phi(x).\end{aligned} \tag{2.6}$$

If we use the general definition

$$\left[\nabla_\mu,\nabla_\nu\right]\phi(x)=\left(\frac{g'}{2}F_{\mu\nu}^{ab}\Sigma_{ab}+g''F_{\mu\nu}I\right)\phi(x) \tag{2.7}$$

and identify the Eqs. (2.6) and (2.7), we obtain:

$$F_{\mu\nu}^{ab}=\partial_\mu A_\nu^{ab}-\partial_\nu A_\mu^{ab}+g'\left(A_{c\mu}^{a}A_\nu^{cb}-A_{c\nu}^{a}A_\mu^{cb}\right), \tag{2.8}$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu \tag{2.9}$$

If we chose $a=i,\ 5$, $b=j,\ 5$, $c=m,\ 5$, with $i,j,m=0,\ 1,\ 2,\ 3$, and denotes $A_\mu^{i5}=2\lambda e_\mu^i$, then the Eq. (2.8) becomes:

$$F_{\mu\nu}^{ij}=\partial_\mu A_\nu^{ij}-\partial_\nu A_\mu^{ij}+g'\left(A_{s\mu}^{i}A_\nu^{sj}-A_{s\nu}^{i}A_\mu^{sj}\right)-4\lambda^2\left(e_\mu^i e_\nu^j-e_\nu^i e_\mu^j\right), \tag{2.10}$$

$$F_{\mu\nu}^{i}=\partial_\mu e_\nu^i-\partial_\nu e_\mu^i+g'\left(A_{s\mu}^{i}e_\nu^s-A_{s\nu}^{i}e_\mu^s\right), \tag{2.11}$$

In a Riemann-Cartan model the quantities $F_{\mu\nu}^i$ are interpreted as the components of the torsion tensor, and $F_{\mu\nu}^{ij}$ as the components of the curvature tensor associated to the gravitational field whose gauge potentials are $e_\mu^i(x)$ and $A_\mu^{ij}(x)$.

3. MODEL WITH SPHERICAL SYMMETRY

We consider now a particular form of spherically gauge fields of the $SO(1, 4) \times U(1)$ group given by the following ansatz:

$$e^0_\mu = (A,0,0,0),\ e^1_\mu = (0,B,0,0),\ e^2_\mu = (0,0,rC,0),\ e^3_\mu = (0,0,0,rC\sin\theta), \tag{3.1}$$

and

$$A^{01}_\mu = (U,0,0,0),\quad A^{12}_\mu = (0,0,W,0),\quad A^{13}_\mu = (0,0,0,Z\sin\theta), \tag{3.2a}$$

$$A^{23}_\mu = (0,0,0,V\cos\theta),\quad A^{02}_\mu = \omega^{03}_\mu = 0, \tag{3.2b}$$

where A, B, C, U, V, Z and W are functions only of the three-dimensional radius r. We use the above expressions to compute the components of the tensors $F^i_{\mu\nu}$ and $F^{ij}_{\mu\nu}$. Their non-null components are:

$$F^0_{10} = A' + g'UB,\quad F^2_{12} = C + rC' - g'WB, \tag{3.3a}$$

$$F^3_{13} = (C + rC' - g'ZB)\sin\theta,\quad F^3_{23} = rC\cos\theta(1 - g'V), \tag{3.3b}$$

and respectively:

$$F^{01}_{10} = U' + 4g'\lambda^2 AB,\quad F^{02}_{20} = g'(UW + 4\lambda^2 rAC), \tag{3.4a}$$

$$F^{03}_{30} = g'\sin\theta\left(UZ + 4\lambda^2 rAC\right),\quad F^{21}_{21} = W' - 4g'\lambda^2 rBC, \tag{3.4b}$$

$$F^{31}_{31} = \left(Z' - 4g'\lambda^2 rBC\right)\sin\theta,\quad F^{32}_{31} = V'\cos\theta, \tag{3.4c}$$

$$F^{31}_{32} = (Z - g'VW)\cos\theta,\quad F^{23}_{32} = \left(V - g'ZW + 4g'\lambda^2 r^2 C^2\right)\sin\theta, \tag{3.4d}$$

where A', C', U', V', W', and Z' denote the derivatives with respect to the variable r.

4. SELF-DUALITY EQUATIONS

In order to obtain the self-duality equations, first of all, we define the dual tensors ${}^*F^{ab}_{\mu\nu}$ and ${}^*F_{\mu\nu}$ by [8, 9]:

$${}^\star F^{ab}_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{\mu\nu\rho\sigma}F^{ab\rho\sigma}, \tag{4.1a}$$

$${}^\star F_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}. \tag{4.1b}$$

Here, “*”, is the Hodge dual map and $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita symbol of rank four, with $\varepsilon_{0123}=1$. We use the spherically symmetric Minkowski metric $g_{\mu\nu}=\left(1,-1,-r^2,-r^2\sin^2\theta\right)$ to rise and lower the space-time indices μ, ν, ρ, $\sigma=1$. The determinant of this metric, appearing in the Eqs. (4.1) has the expression: $g=\det\left(g_{\mu\nu}\right)=-r^4\sin^2\theta$.

Using the definition (4.1a) we obtain the following non-null components of the dual tensors $\star F^i_{\mu\nu}$ and $\star F^{ij}_{\mu\nu}$:

$$\star F^3_{20}=ABC(C+rC'-g'ZB),\quad \star F^2_{03}=ABC\left(C+rC'-g'WB\right)\sin\theta, \tag{4.2a}$$

$$\star F^3_{01}=\frac{ABC^2\left(1-g'V\right)}{r}\cot\theta,\quad \star F^0_{23}=r^2ABC\left(A'+g'UB\right)\sin\theta, \tag{4.2b}$$

and respectively

$$\star F^{31}_{10}=\frac{ABC(Z-g'VW)}{r^2\sin\theta},\quad \star F^{23}_{10}=\frac{ABC(V-g'ZW+4g'\lambda^2r^2C^2)}{r^2}, \tag{4.3a}$$

$$\star F^{31}_{20}=ABC(Z'-4g'\lambda^2rBC),\quad \star F^{21}_{30}=ABC\left(W'-4g'\lambda^2rBC\right)\sin\theta, \tag{4.3b}$$

$$\star F^{30}_{21}=g'ABC(ZU+4\lambda^2rAC),\quad \star F^{02}_{31}=g'ABC\left(WU+4\lambda^2rAC\right)\sin\theta, \tag{4.3c}$$

$$\star F^{10}_{32}=r^2ABC(U'+4g'\lambda^2BA)\sin\theta,\quad {}^*F^{32}_{02}=ABCV'\cot\theta. \tag{4.3d}$$

We will suppose that the gravitational field has spherical symmetry and it is created by a point-like mass m (the source). In the same time, we will admit that the electromagnetic field $A_\mu(x)$ is due to a constant electric charge Q of the same source, *i.e.*, we will consider that the point-like mass m is also electrical charged. The condition of self-duality for the electromagnetic tensor field $F_{\mu\nu}$ means, in fact, the duality between the electric and magnetic fields. The duality transformation changes the field of an electrical charge in the magnetic one (of a monopole) and the inverse map is also satisfied. This is a well known property (the duality) of the electromagnetic field. The only non-null components of the electromagnetic strength tensor field are the following:

$$F_{01}=-F_{10}=-\frac{Q}{4\pi\varepsilon_0r^2}. \tag{4.4}$$

This form will be used in Sect. 5 to calculate the energy-momentum tensor of the electromagnetic field and to write the Yang-Mills field equations.

The self-duality equations for the gravitational field have the general form [9]:

$$^{*}F^{i}_{\mu\nu} = iF^{i}_{\mu\nu}, \tag{4.5a}$$

$$^{*}F^{ij}_{\mu\nu} = iF^{ij}_{\mu\nu} \tag{4.5b}$$

Introducing the expressions (4.2a)–(4.2b) into the duality equations (4.5a), we obtain the following field equations:

$$g'V = 1, \tag{4.6a}$$

$$C + rC' - g'BZ = 0, \tag{4.6b}$$

$$C + rC' - g'BW = 0, \tag{4.6c}$$

$$A' + g'BU = 0. \tag{4.6d}$$

The Eqs. (4.6b) and (4.6c) impose the condition $Z = W$, and the Eq. (4.6a) gives the solution $V(r) = \frac{1}{g'}$ for this gauge potential. Then, introducing the expressions (4.3a)–(4.3d) into the duality condition (4.5b), we obtain the second set of independent self-duality equations:

$$U' + 4g'\lambda^2 AB = 0, \tag{4.7a}$$

$$Z' - 4g'\lambda^2 rBC = 0, \tag{4.7b}$$

$$UW + 4\lambda^2 rAC = 0. \tag{4.7c}$$

We will search the solutions of the Eqs. (4.6)–(4.7) satisfying the conditions: $B(r) = \frac{1}{A(r)}$, $C(r) = 1$. Then, we obtain $Z(r) = \frac{1}{g'B(r)} = \frac{A(r)}{g'} =$ $= W(r)$, and the only independent self-duality equations are:

$$U' + 4g'\lambda^2 = 0, \tag{4.8a}$$

$$Z' - \frac{4g'\lambda^2 r}{A} = 0, \tag{4.8b}$$

$$A' + \frac{g'U}{A} = 0. \tag{4.8c}$$

The solution of the Eq. (4.8a) is the following:

$$U(r) = -4g'\lambda^2 r + \alpha, \tag{4.9}$$

where α is an arbitrary constant of integration. For simplicity, we will chose $\alpha = 0$, and then (4.9) becomes

$$U(r) = -4g'\lambda^2 r. \tag{4.10}$$

The Eq. (4.8c) can be written then in the form:

$$AA' - 4g'\lambda^2 r = 0, \tag{4.11}$$

or, equivalently:

$$\frac{1}{2}\frac{\mathrm{d}(A^2)}{\mathrm{d}r} - 4g'\lambda^2 r = 0. \tag{4.12}$$

Integrating this equation, we obtain the solution for the gauge potential $A(r)$:

$$A^2(r) = 4g'\lambda^2 r^2 + \beta, \tag{4.13}$$

where β is another constant of integration. If we chose the value $\beta = 1$, then the gauge potential (4.13) can be written finally as:

$$A(r) = \sqrt{1 - \frac{\Delta}{3} g'^2 r^2}, \tag{4.14}$$

where we introduced the cosmological constant $\Lambda = -12\lambda^2$.

The result (4.14) is known as de-Sitter solution with the cosmological constant determined by the deformation parameter λ of the $SO(1,4)$ group. It includes also the gravitational gauge coupling constant g'.

5. THE YANG-MILLS FIELD EQUATION

The Yang-Mills field equation for the gravitational potentials $e^i_\mu(x)$ can be obtained by imposing the variational principle $\delta_e S = 0$ to the total integral of action associated to the system composed of the two fields (gravitational and electromagnetic) [10]. They have the form:

$$F^i_\mu - \frac{1}{2} F e^i_\mu = 8\pi G T^i_\mu, \tag{5.1}$$

where F^i_μ and F are defined by:

$$F^i_\mu = F^{ij}_{\mu\nu} \bar{e}^\mu_i, \quad F = F^{ij}_{\mu\nu} \bar{e}^\mu_i \bar{e}^\nu_j, \tag{5.2}$$

and T^i_μ is the energy-momentum tensor of the electromagnetic field. In the Eq. (5.2) we denoted by $\bar{e}^\mu_i$ the inverse of e^i_μ satisfying the usual properties [6]:

$$e^i_\mu \bar{e}^\mu_j = \delta^i_j, \quad e^i_\mu \bar{e}^\nu_i = \delta^\nu_\mu, \tag{5.3}$$

In the same way, we can show that the field equations for the other gravitational gauge potentials, $A^{ij}_\mu(x)$, are equivalent with [11]:

$$F^i_{\mu\nu} = 0. \tag{5.4}$$

Writing the Eqs. (5.1) and (5.4) for the ansatz (3.1)–(3.2), we obtain the following independent Yang-Mills equations:

$$-\frac{2AA'}{r} + \frac{1-A^2}{r^2} + 12\lambda^2 = g'^2 \frac{Q^2}{r^4}, \tag{5.5a}$$

$$-\frac{2AA'}{r} + U' + 12\lambda^2 = -g'^2 \frac{Q^2}{r^4}, \tag{5.5b}$$

and respectively

$$AA' + U = 0. \tag{5.5c}$$

The two equations (5.5a) and (5.5b) are compatible if we impose the condition:

$$\frac{1-A^2}{r^2} - U' = \frac{2g'^2Q^2}{r^4}. \tag{5.6}$$

Introducing (5.5c) into the Eq. (5.6) and denoting $A^2 = y$, we obtain the following differential equation for the new unknown function $y(r)$:

$$r^2 y'' - 2y + 2 = \frac{4g'^2Q^2}{r^4}. \tag{5.7}$$

The solution of the differential equation (5.7), which is of second order in the derivatives of the function $y(r)$ with respect to the variable r, is:

$$y(r) = A^2(r) = 1 + g'^2\left(\frac{\alpha}{r} + \frac{Q^2}{r^2} + \beta r^2\right), \tag{5.8}$$

where α and β are constants of integration. It is known [12] that the constant α is determined by the mass m of the point-like source that creates the gravitational field: $\alpha = -2m$. The other constant β is determined by condition that the solution (5.8) verifies the Eqs. (5.5a)–(5.5b) that now coincide as a consequence of the constraint (5.6). Introducing the solution (5.8) into the Eq. (5.5a), we obtain:

$$\beta = 4\lambda^2 = -\frac{\Lambda}{3}, \quad \Lambda = -12\lambda^2. \tag{5.9}$$

Finally, the solution of the field equations (5.5a)–(5.5c) is:

$$A(r)=\sqrt{1-g'^2\left(\frac{2m}{r}-\frac{Q^2}{r^2}+\frac{\Lambda}{3}r^2\right)},\quad U=-g'^2\left(\frac{m}{r}-\frac{Q^2}{r^3}-\frac{\Lambda}{3}r\right). \tag{5.10}$$

If we consider the contraction $\lambda\to 0$, then the de-Sitter group $SO(1,4)$ becomes the Poincaré group $SO(1,3)$ and the solution (5.10) reduces to the Reissner-Nordström one. Moreover, comparing (5.10) with (4.14) we can see that the Reissner-Nordström solution is not a self-dual one. However, as we shown in this section, the Yang-Mills equations have both de-Sitter and Reissner-Nordström as their solutions.

6. CONCLUDING REMARKS

We have studied an unified self-dual gauge theory of gravitational and electromagnetic field using the group $SO(1,4)\times U(1)$ as local symmetry. The base space-time were chosen to be of Minkowski type and its geometrical structure remained rigged, *i.e.*, it has not been affected by physical events.

A model of gauge theory with spherical symmetry was constructed supposing that the gauge potentials depend only of the $3D$ radius r of the Minkowski space-time. The self-duality equations have been written and their solutions studied. We proven that these equations admit the de-Sitter solution, but no the Reissner-Nordström one.

To compare the self-duality solutions with other known results we studied also the Yang-Mills field equations. We have established that these equations admit both de-Sitter and Reissner-Nordström as their solutions.

We emphasizes that the model developed in this paper can be applied to the study of electromagnetic and gravitational interactions between different matter fields.

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