

THE ENERGY DISTRIBUTION OF A STRINGY MAGNETIC BLACK HOLE

IRINA RADINSCHI*

Department of Physics, "Gh. Asachi" Technical University, Iași, 6600, Romania

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In this paper we calculate the energy distribution of a magnetic stringy black hole solution in the Einstein prescription. The energy distribution depends on the mass M and charge Q of the black hole. Our result shows that the total energy is dependent on charge Q and differs to previous investigations.

1. INTRODUCTION

It is well-known that one of the most interesting and challenging problems of general relativity is the energy and momentum localization. There are many attempts to evaluate the energy distribution in a general relativistic system. One of the methods used for the energy and momentum localization is the one which used the energy-momentum complexes. After the Einstein work [1, 2], a large number of definitions for the energy distribution was given. We mention the expressions proposed by Landau and Lifshitz [3], Papapetrou [4], Bergmann [5], Weinberg [6] and Møller [7]. The Einstein, Landau and Lifshitz, Papapetrou, Bergmann and Weinberg energy-momentum complexes (ELLPBW) are restricted to calculate the energy distribution in quasi-Cartesian coordinates.

Related on the method that used the energy-momentum complexes we can say that there are doubts that these prescriptions could give acceptable results for a given space-time. The problem is that with different energy-momentum complexes we can obtain different expressions for the energy associated with a given space-time. This is because most of the energy-momentum complexes are restricted to the use of particular coordinates. But some interesting results obtained by several authors [8, 9] demonstrated that the energy-momentum complexes are good tools for evaluating the energy and momentum in general relativity.

In this paper we calculate the energy distribution of the dual solution in the string frame that is known as the magnetic stringy black hole solution. We perform the calculations in the Einstein prescription. We study the energy associated with this solution because we think it can furnish us an interesting

* iradinsc@phys.tuiasi.ro

result. The metric is obtained by multiplying the electric metric in the Einstein frame by a factor $e^{-2\Phi}$. Through the paper we use geometrized units ($G = 1$, $c = 1$) and follow the convention that Latin indices run from 0 to 3.

2. ENERGY IN THE EINSTEIN PRESCRIPTION

The low energy effective theory largely resembles general relativity with some new ‘‘matter’’ fields as the dilaton, axion etc [10, 11]. A main property of the low-energy theory is that there are two different frames in which the features of the space-time may look very different. These two frames are the Einstein frame and the string frame and they are related to each other by a conformal transformation ($g_{\mu\nu}^E = e^{-2\Phi} g_{\mu\nu}^S$) which involves the massless dilaton field as the conformal factor. The string ‘‘sees’’ the string metric. Many of the important symmetries of string theory also rely of the string frame or the Einstein frame [12].

The action for the Einstein-dilaton-Maxwell theory is given by

$$S_{EDM} = \int d^4x \sqrt{-g} e^{-2\Phi} \left[R + 4 g_{\mu\nu} \nabla^\mu \Phi \nabla^\nu \Phi - \frac{1}{2} g^{\mu\lambda} g^{\nu\rho} F_{\mu\nu} F_{\lambda\rho} \right]. \quad (1)$$

Varying with respect to the metric, dilaton and Maxwell fields we get the field equations for the theory given as

$$R_{\mu\nu} = -2 \nabla_\mu \Phi \nabla_\nu \Phi + 2 F_{\mu\lambda} F_\nu^\lambda, \quad (2)$$

$$\nabla^\nu (e^{-2\Phi} F_{\mu\nu}) = 0, \quad (3)$$

$$4 \nabla^2 \Phi - 4 (\nabla \Phi)^2 + R - F^2 = 0. \quad (4)$$

The metric (in the string frame) which solve the Einstein-dilaton-Maxwell field equations to yield the electric black hole is given by

$$ds^2 = A \left(1 + \frac{2M \sinh^2 \alpha}{r} \right)^{-2} dt^2 - \frac{1}{A} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2. \quad (5)$$

where $A = 1 - \frac{2M}{r}$.

In the string frame the dual solution known as the magnetic black hole is obtained by multiplying the electric metric in the Einstein frame by a factor $e^{-2\Phi}$. Therefore, the magnetic black hole metric is given by

$$ds^2 = \frac{A}{B} dt^2 - \frac{1}{AB} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (6)$$

with $B = 1 - \frac{Q^2}{Mr}$.

The Einstein energy-momentum complex [1, 2] is given by

$$\Theta_i^k = \frac{1}{16\pi} H_{i,l}^{kl}, \quad (7)$$

where

$$H_i^{kl} = -H_i^{lk} = \frac{g_{in}}{\sqrt{-g}} [-g(g^{kn}g^{lm} - g^{ln}g^{km})]_{,m}. \quad (8)$$

Θ_0^0 and Θ_α^0 are the energy and, respectively, the momentum density components.

The Einstein energy-momentum complex satisfies the local conservation laws

$$\frac{\partial \Theta_i^k}{\partial x^k} = 0. \quad (9)$$

Integrating Θ_i^k over the three-space gives the energy and momentum components

$$P_i = \iiint \Theta_i^0 dx^1 dx^2 dx^3. \quad (10)$$

P_0 is the energy and P_α are the momentum components.

Using the Gauss theorem we obtain

$$P_i = \frac{1}{16\pi} \iint H_i^{0\alpha} n_\alpha dS, \quad (11)$$

where $n_\alpha = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right)$ are the components of a normal vector over an infinitesimal surface element $dS = r^2 \sin \theta d\theta d\varphi$.

For making the calculations using the Einstein energy-momentum complex we transform the metric given by (6) to quasi-Cartesian coordinates t, x, y, z according to $x = r \sin \theta \cos \varphi$, $y = r \sin \theta \sin \varphi$ and $z = r \cos \theta$.

Using (7), (8), (10) and applying the Gauss theorem we obtain that the energy distribution of the magnetic black hole is given by

$$E(r) = \frac{1}{2} \frac{rQ^2 + 2M^2r - 2MQ^2}{(Mr - Q^2)}. \quad (12)$$

The energy distribution depends on the mass M and charge Q . After some calculations we obtain for the energy

$$E(r) = M - \frac{1}{2}r + \left(-\frac{1}{2} \frac{M}{Q^2} \right) r^2 + \left(-\frac{1}{2} \frac{M^2}{Q^4} \right) r^3 + \left(-\frac{1}{2} \frac{M^3}{Q^6} \right) r^4 + O(r^5). \quad (13)$$

3. DISCUSSION

The subject of the localization of energy continues to be an open one since Einstein has given his important result of the special theory of relativity that mass is equivalent to energy. Misner *et al.* [13] sustained that to look for a local energy-momentum means that is looking for the right answer to the wrong question. Also, they concluded that the energy is localizable only for spherical systems. On the other hand, Cooperstock and Sarracino [14] demonstrated that if the energy is localizable in spherical systems then it is also localizable in any space-times. Bondi [15] gave his opinion that “a nonlocalizable form of energy is not admissible in general relativity, because any form of energy contributes to gravitation and so its location can in principle be found”.

The problem of the energy and momentum localization was re-opened by Virbhadra and his collaborators, and since then, many interesting results were obtained in this area [8, 9]. The problem of the energy localization with the energy-momentum complexes was extended to 2 and 3 dimensional space-times [8, 9]. It was demonstrated that for these cases the (ELLPBW) and Møller energy-momentum complexes allow us to obtain interesting results. Also, Chang, Nester and Chen [16] showed that the energy-momentum complexes are actually quasilocal and legitimate expression for the energy-momentum. They concluded that there exist a direct relationship between energy-momentum complexes and quasilocal expressions because every energy-momentum complexes is associated with a legitimate Hamiltonian boundary term.

Even the method of localization of energy with several energy-momentum complexes has many adepts there was, also, many criticism related to the use of the energy-momentum complexes in energy and momentum localization. The main lack of the energy-momentum complexes is that most of these restrict one to calculate in quasi-Cartesian coordinates.

In this paper we choose to compute the energy associated with a magnetic stringy black hole solution because this solution is interesting to study. The energy of the magnetic black hole, which is the dual solution in the string frame, depends on the mass M and charge Q of the black hole. Our result shows that the total energy is dependent on charge Q and differs to previous investigations. We evaluate the energy distribution in Schwarzschild Cartesian coordinates. It will be interesting, in a future work, to compare this result with those obtained in Schwarzschild Cartesian coordinates and Kerr-Schild Cartesian coordinates in the Einstein, Landau and Lifshitz, Papapetrou and Weinberg prescriptions.

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