SOLUTIONS OF THE DE-SITTER GAUGE THEORY

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The de-Sitter gauge theory with SO(1,4) as symmetry group is considered. A real parameter that determines a deformation of the Lie algebra of this group is introduced. It determines the cosmological constant of the model. When this parameter vanishes we obtain the Poincairé gauge theory of the gravitational field without cosmological constant. Solutions of the gravitational field equations are obtained considering a model with spherically symmetric gauge potentials. In particular, the case of gravitational field created by a point-like mass having a constant electric charge is considered. The duality property is also studied considering a magnetic monopole with non-null mass as source of the gravitational field. Our de-Sitter gauge theory of gravitational has the Minkowski space-time as base manifold and its geometrical structure is not affected anymore by the gauge transformations.

1. INTRODUCTION

The gauge theory of gravitation has been considered by many authors in order to describe the gravity in a similar way with other interactions (electromagnetic, weak or strong) [1]. Some authors consider the Poincairé group (*PG*) or de-Sitter (*DS*) group as "active" symmetry groups, *i.e.*, acting on the space time coordinates [2]. Others adopt the "passive" point of view when the space-time coordinates are not affected by group transformations [3, 4]. Only the fields change under the action of the symmetry group.

Although the Poincaré gauge theory leads to a satisfactory classical theory of gravity, the analogy with gauge theories of internal symmetries is not a satisfactory one because of the specific treatment of translations [5]. It is possible, however, to formulate the gauge theory of gravity in a way that treats the whole *PG* in a more unified framework. The approach is based on the *DS* group and the Lorentz and translation parts are distinguished through a mechanism of spontaneous symmetry breaking [6]. An immediate consequence of replacing *PG* by the *DS* group as the symmetry underlying the Universe is the appearance of a non-vanishing cosmological constant Λ, which is determined by a real parameter λ of deformation. When we consider the limit $\lambda \to 0$, *i.e.* the group contraction process, the *DS* group reduces to the *PG*, and the corresponding gravitation theory can not describe the cosmological constant [7]. The matter fields are described by an action that is invariant under the global *DS* symmetry and the gravity is introduced as a gauge field in the process of localization of this symmetry.

In this work, we adopt the "passive" point of view for the symmetry group in order to develop a *DS* gauge theory of gravitation over a spherical symmetric Minkowski space-time. Therefore, we restrict ourselves to recast *DS* symmetry and its consequences used to specify the space-time events is no longer affected by *DS* transformations.

Section 2 is devoted to the formulation of the *DS* gauge model on a spherical Minkowski space-time. The general expressions for the components $F^{A}_{\mu\nu}(x)$ of the strength tensor of the gauge fields are obtained. A particular ansatz for the gauge fields is chosen and the corresponding components, $F^{A}_{\mu\nu}(x)$, are presented in Section 3.

In the following three Sections we give different types of solutions within the framework of *DS* gauge theory of gravity. Thus, the Section 4, is devoted to obtaining the Schwarzschild-de-Sitter type solution. A constraint is imposed such that the gauge field equations be compatible. In Section 5, we show how can be obtained a solution without singularities of the *DS* gauge field equations. Some constraints on the invariants of the model are imposed and an example of such solution is given. Its dependence on the cosmological constant is also studied. Finally, in Section 6 we consider the case when the gravitational field is created by a point-like source of mass *m* which has also an electrical charge *Q*. The duality property is also studied considering a magnetic monopole with non-null mass as source of the gravitational field.

The conclusion is that the *DS* group can be considered as a "passive" gauge symmetry group for gravitation. Therefore, the gravitation can be described by gauge potentials defined on a Minkowski space-time and we do not have to use Riemann or Riemann-Cartan theories.

2. THE de-SITTER GAUGE THEORY

We develop a gauge theory of the *DS* group in a four-dimensional Minkowski space-time M_4, endowed with spherical symmetry:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right). \tag{2.1}$$

The *DS* group is 10-dimensional and its infinitesimal generators are denoted by Π_a, and $M_{ab} = -M_{ba}$, $a = 0, 1, 2, 3$, [8, 9]. In the order to give a general formulation of the gauge theory for the *DS* group, we denote these generators as X_A, A = 1, 2, …, 10. The equations of structures can then be written under the general form:

$$\left[X_A, X_B\right] = if_{AB}^C X_C, \tag{2.2}$$

where $f_{AB}^C = -f_{BA}^C$ are the constants of structure whose concrete expressions will be given below [see Eq. (2.5)].

Let us suppose now that the *DS* group is a gauge group for gravitation; correspondingly, we introduce ten gauge fields: $h_\mu^A(x)$, A = 1, 2, …, 10, μ = 0, 1, 2, 3. We then construct the tensor of the gauge fields (strength tensor), $F_{\mu\nu} = F_{\mu\nu}^A X_A$, which takes its values in the Lie algebra of the *DS* group (Lie algebra – valued tensor). The components of this tensor are given by:

$$F_{\mu\nu}^A = \partial_\mu h_\nu^A - \partial_\nu h_\mu^A + f_{BC}^A h_\mu^B h_\nu^C. \tag{2.3}$$

In order to write the constants of structure, we use the following notation for the index A:

$$A = \begin{cases} a = 0, 1, 2, 3, \\ [ab] = [01], [02], [03], [12], [13], [23]. \end{cases} \tag{2.4}$$

This means that A can stand for a single index like 2 or 3, as well as for a pair of indices like [01], [12], etc. The infinitesimal generators X_A of the DS group are interpreted as: $X_a = \Pi_a$ (the de-Sitter "translations" operators) and $X_{[ab]} = M_{ab}$ (the Lorentz transformation operators) with the property $M_{ab} = -M_{ba}$. The constants of structure have the following expressions [7]:

$$f_{bc}^a = f_{c[de]}^{[ab]} = f_{[bc][de]}^a = f_{cd}^{[ab]} = 0, \tag{2.5a}$$

$$f_{cd}^{[ab]} = 4\lambda^2\left(\delta_c^b\delta_d^a - \delta_c^a\delta_d^b\right) = -f_{dc}^{[ab]} \tag{2.5b}$$

$$f_{b[cd]}^a = -f_{[cd]b}^a = \frac{1}{2}\left(\eta_{bc}\delta_d^a - \eta_{bd}\delta_c^a\right), \tag{2.5c}$$

$$f_{[ab][cd]}^{[ef]} = \frac{1}{4}\left(\eta_{bc}\delta_a^e\delta_d^f - \eta_{ac}\delta_b^e\delta_d^f + \eta_{ad}\delta_b^e\delta_c^f - \eta_{bd}\delta_a^e\delta_c^f\right) - e \longleftrightarrow f \tag{2.5d}$$

Here, λ is a deformation parameter, and $\eta_{ab} = (1,-1,-1,-1)$ is the Minkowski metric in the tangent space to M_4. When $\lambda \to 0$, we obtain the Poincairé-Lie algebra.

We denote the gauge fields $h_\mu^A(x)$, by $e_\mu^a(x)$ (tetrad fields) if $A = a$, and by $A_\mu^{ab}(x) = -A_\mu^{ba}(x)$ (spin connection) if $A = [ab]$. Then, introducing the relations (2.5a)–(2.5d), into the definition, Eq. (2.3), we find the expressions of the strength tensor components:

$$F^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu + \left(A^{ab}_\mu e^c_\nu - A^{ab}_\nu e^c_\mu\right)\eta_{bc}, \tag{2.6}$$

$$F^{ab}_{\mu\nu} = \partial_\mu A^{ab}_\nu - \partial_\nu A^{ab}_\mu + \left(A^{ac}_\mu A^{db}_\nu - A^{ac}_\nu A^{db}_\mu\right)\eta_{cd} - 4\lambda^2\left(e^a_\mu e^b_\nu - e^a_\nu e^b_\mu\right). \tag{2.7}$$

In a Riemann-Cartan theory, the quantity $F^a_{\mu\nu}$ is interpreted as the torsion tensor, and $F^{ab}_{\mu\nu}$ as the curvature tensor of space-time defined by the gravitational gauge fields $e^a_\mu(x)$ and $A^{ab}_\mu(x)$.

3. THE EQUATIONS OF THE GAUGE FIELDS

We consider a particular form of spherically gauge fields of the DS group given by the following ansatz:

$$e^0_\mu = (A, 0, 0, 0), \quad e^1_\mu = \left(0, \frac{1}{A}, 0, 0\right), \tag{3.1a}$$

$$e^2_\mu = (0, 0, rC, 0), \quad e^3_\mu = (0, 0, 0, rC\sin\theta), \tag{3.1b}$$

$$A^{01}_\mu = (U, 0, 0, 0), \quad A^{12}_\mu = (0, 0, W, 0), \quad A^{13}_\mu = (0, 0, 0, Z\sin\theta), \tag{3.2a}$$

$$A^{23}_\mu = (V, 0, 0, \cos\theta), \quad A^{02}_\mu = A^{03}_\mu = (0, 0, 0, 0), \tag{3.2b}$$

where A, C, U, V, W and Z are functions only of the three-dimensional radius r. We use the above expressions to compute the components of the tensor $F^a_{\mu\nu}$ and $F^{ab}_{\mu\nu}$. The field equations for the gauge fields $e^a_\mu(x)$ in vacuum have the form [7]:

$$F^a_\mu - \frac{1}{2}Fe^a_\mu = 0, \tag{3.3}$$

where

$$F^a_\mu = F^{ab}_{\mu\nu}\,\bar{e}^\nu_b, \quad F = F^{ab}_{\mu\nu}\,\bar{e}^\mu_a\,\bar{e}^\nu_b, \tag{3.4}$$

with $\bar{e}^\mu_a$ denoting, as usually, the inverse of e^a_μ. If we consider the model with a null strength tensor $F^a_{\mu\nu}$ and $C = 1$, then from Eq. (3.3) we obtain only two independent equations

$$-\frac{2AA'}{r} + \frac{1 - A^2}{r^2} + 12\lambda^2 = 0, \tag{3.5a}$$

$$-\frac{2AA'}{r}+U'+12\lambda^2=0, \tag{3.5b}$$

4. SOLUTION OF SCHWARZCHILD – DS TYPE

The equations (3.5a) and (3.5b) are compatible if we choose the function $A(r)$, so that

$$U'=\frac{1-A^2}{r^2}. \tag{4.1}$$

Taking into account that $F^a_{\mu\nu}=0$ imposes $U=-AA'$, the condition (4.1) becomes:

$$r^2(A^2)''-2A^2+2=0. \tag{4.2}$$

The solution of Eq. (4.2) is

$$A^2=1+\frac{\alpha}{r}+\beta r^2. \tag{4.3}$$

where α and β are two arbitrary constants. This solution also verifies the field equations (3.5), if and only if $\beta=4\lambda^2$. But according to the result of MacDowell-Mansouri [9], the cosmological constants of the model is identified as $\Lambda=-12\lambda^2$. The solution (4.3) takes then the form:

$$A^2=1+\frac{\alpha}{r}-\frac{\Lambda}{3}r^2. \tag{4.4}$$

In particular, if we choose $\alpha = -2m$, then we obtain the Schwarzchild-de-Sitter solution:

$$A^2=1-\frac{2m}{r}-\frac{\Lambda}{3}r^2. \tag{4.5}$$

In the limit $\lambda \to 0$, we obtain the Schwarzschild solution:

$$A^2=1-\frac{2m}{r}. \tag{4.6}$$

For $\alpha = 0$, the solution (4.4) is that of de-Sitter.

The potentials $A^{ab}_\mu(x)$ can be expressed as functions of $e^a_\mu(x)$ because of the constraints $F^a_{\mu\nu}=0$. In the corresponding Riemannian model these constraints are equivalently with vanishing of torsion for the space-time. Therefore, the gauge fields $e^a_\mu(x)$ can be interpreted as the tetrad fields of the gravitational field. The spin connection components $A^{ab}_\mu(x)$ are determined, as we mentioned above, by tetrads $e^a_\mu(x)$ (*i.e.* they are not independent fields):

$$U=-\frac{m}{r^2}+\frac{\Lambda}{3}r, \quad Z=W=\sqrt{1-\frac{2m}{r}-\frac{\Lambda}{3}r^2}. \tag{4.7}$$

The previous results show that the model presented here can be considered a gauge theory for the "active" *DS* symmetry group.

5. SOLUTIONS WITHOUT SINGULARITIES

Using the model given in the previous Section, we can write an integral of action, quadratic in the components . Then, we can impose some constraints for non-singular solutions of the field equations. These constraints are introduced in the expression of integral action by means of two Langrange-multiplier fields $\varphi_1(t)$ and $\varphi_2(t)$.

The integral of action associated to the gravitational gauge fields $e_\mu^a(x)$ and $A_\mu^{ab}(x)$ has the form [10]:

$$S_g=\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^A F_{\rho\sigma}^B Q_{AB}, \tag{5.1}$$

where $\varepsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita symbol of rank four, with $\varepsilon^{0123}=1$. This action is independent of any specific metric $g_{\mu\nu}$ on M_4; indeed, the property of general covariance for action imposes the volume element $\sqrt{-g}\,\mathrm{d}^4x$, [with $g=\det(g_{\mu\nu})$] and the tensor Levi-Civita has the form $\frac{1}{\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}$, so that the $g_{\mu\nu}$-dependence of S_g cancels.

The quantities Q_{AB} are constants symmetric with respect to the indices $Q_{AB}=-Q_{BA}$. If we choose [9]

$$Q_{AB}=\begin{cases}\varepsilon_{abcd}, & \text{for } A=[ab], B=[cd],\\ 0, & \text{otherwise,}\end{cases} \tag{5.2}$$

then we obtain the action of the General Relativity (*GR*). It is possible also to obtain the integral of Teleparallel Gravity (*TG*) by an appropriate choice of Q_{AB} [11, 12].

Now, we use the form given in Eq. (5.2) in order to obtain solutions without singularities of *DS*-gauge theory of gravitation. Namely, we impose restrictions [13] on two invariants I_1 and I_2 [see Eq. (5.6)] of the theory. Introducing the Langrange-multipliers $\varphi_1(t)$ and $\varphi_2(t)$ and, and using the choice (5.2), the integral of action (5.1) can be written as:

$$S_g = -\frac{1}{16\pi G}\int \mathrm{d}^4 x e\left[F + \varphi_1(t) f_1(I_1) + \varphi_2(t) f_2(I_2) + V(\varphi_1, \varphi_2)\right], \tag{5.3}$$

where $e = \det\left(e_\mu^a\right)$. The quantities $f_i(I_i)$, $i = 1, 2$ are two functions which must be chosen in an appropriate form in order to obtain solutions without singularities of the corresponding field equations. Thus, the potential $V(\varphi_1, \varphi_2)$ have to satisfy the constraint equations [10, 13]:

$$f_1(I_1) = -\frac{\partial V}{\partial \varphi_1}, \quad f_2(I_2) = -\frac{\partial V}{\partial \varphi_2}, \tag{5.4}$$

As an example, we chose the function $f_i(I_i)$ in the simple form [13]:

$$f_1(I_1) = I_1, \quad f_2(I_2) = -\sqrt{I_2}, \quad V(\varphi_1, \varphi_2) = V_1(\varphi_1) + V_2(\varphi_2), \tag{5.5}$$

with

$$I_1 = F - \sqrt{3}\left(4F_\mu^a F_a^\mu - F^2\right)^{1/2}, \quad I_2 = 4F_\mu^a F_a^\mu - F^2. \tag{5.6}$$

We emphasize here that the indices a, b, c… are raised and lowered with the metric η_{ab}, and the indices μ, ν, ρ, σ by the Minkowski metric of the space-time. In addition, we chose a particular form of spherically gauge fields $e_\mu^a(x)$ and $A_\mu^{ab}(x)$ corresponding to the Robertson Walker metric with (as usually) $a(t)$ a function only of time variable, k – a constant, and $a' = \frac{\mathrm{d}a}{\mathrm{d}t}$. Then the field equations for $a(t)$ and the multiplier fields chosen as $\varphi_1(t) = 0$ and $\varphi_2(t) = \varphi(t)$ are:

$$\begin{aligned} H' &= -\frac{1}{2\sqrt{3}}\frac{\mathrm{d}V}{\mathrm{d}\varphi}, \\ \varphi' &= -3H\varphi + \sqrt{3}H - \frac{1}{2\sqrt{3}H}V - \frac{2\Lambda}{\sqrt{3}H}. \end{aligned} \tag{5.7}$$

We consider the potential $V(\varphi)$ of the simple form:

$$V(\varphi) = 2\sqrt{3}\lambda^2\left(\frac{\varphi^2}{1+\varphi^2} + 8\sqrt{3}\right), \tag{5.8}$$

where λ is the real parameter that determines the cosmological constant Λ. This parameter coincides with the constant H_0 in Ref. [13] that has been interpreted as a Planck scale of the model. Therefore, in our example the Planck scale is related to the cosmological constant Λ. For small values of H and φ, and with the potential (5.8), the Eqs. (5.7) can be written as:

$$H' \simeq -2\lambda^2\varphi, \qquad \varphi'(t) \simeq \frac{\sqrt{3}H^2 - \lambda^2\varphi^2}{H}. \tag{5.9}$$

These equations have the periodic solution:

$$\varphi(t) = \varphi_0 \sin(\omega t), \quad H(t) = \frac{\omega\varphi_0}{2\sqrt{3}}\left[\cos(\omega t) - 1\right], \tag{5.10}$$

where φ_0 is an integration constant and $\omega = 2\sqrt[4]{3}\lambda$ is the frequency of oscillation of the corresponding gravitational described by the gauge potentials $e_\mu^a(x)$ and $A_\mu^{ab}(x)$. This solution has no singularities and it is valid if the cosmological constant is negative ($\Lambda < 0$). The case with positive cosmological constant ($\Lambda > 0$) can be studied choosing the anti-de-Sitter group as gauge group. But, the deformation parameter λ will be then pure imaginary. We emphasize that there are possible also periodic solutions if we suppose a time-dependent cosmological "constant". In particular, we can consider a cosmological "constant" which is itself a periodic function on time. It will be also of interest to apply the previous method in obtaining non-singular solutions of the gauge theories with internal groups of symmetry.

6. THE REISSNER-NORDSTRÖM SOLUTION OF THE YANG-MILLS EQUATIONS

We will apply the previous *DS* gauge theory to the case when the source of the gravitational field is a point-like mass *m* that has also a constant electric charge *Q*. Therefore, this source creates also an electromagnetic field $A_\mu(x)$. Its corresponding tensor field is:

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu; \tag{6.1}$$

the only non-null its components are [14]:

$$A_{10} = -A_{01} = -\frac{Q}{4\pi\varepsilon_0 r^2}. \tag{6.2}$$

The field equations for the gauge fields $e_\mu^a(x)$ (gravitational) and $A_\mu(x)$ (electromagnetic) have the same form as those of Eq. (3.3) but with the corresponding energy-momentum T_μ^a of the electromagnetic field on the right-hand side:

$$F_\mu^a - \frac{1}{2}Fe_\mu^a = 8\pi G T_\mu^a. \tag{6.3}$$

The tensor T_μ^a is defined by the relation [14]:

$$T_\mu^a = \frac{1}{K}\left(A_\nu^a A_\mu^b \overline{e}_b^\nu - \frac{1}{4} A_\nu^b A_b^\nu e_\mu^a \right), \tag{6.4}$$

where K is a constant that will be chosen in a convenient form to simplify the solutions of the field equations. It can be shown that the field equations for the other gravitational potentials, $A_\mu^{ab}(x)$ are equivalent with [9]:

$$F_{\mu\nu}^a = 0. \tag{6.5}$$

We consider now a particular form of spherically gravitational field, given by the relations (3.1)–(3.2) with $C = 1$ and $V = W = Z = 0$. Then, using the Eq. (6.5) and choosing the constant K in (6.4) so that

$$\frac{G}{4\pi K g'^2 \varepsilon_0^2} = 1, \tag{6.6}$$

the field equations (6.3) for $e_\mu^a(x)$ field are [15]:

$$-\frac{2AA'}{r} + \frac{1 - A^2}{r^2} + 12\lambda^2 = g'^2 \frac{Q^2}{r^4}, \tag{6.7a}$$

$$-\frac{2AA'}{r} + U' + 12\lambda^2 = g'^2 \frac{Q^2}{r^4}. \tag{6.7b}$$

Here g' denotes the coupling constant for the gravitational field. These equations are compatible if we impose the condition:

$$\frac{1 - A^2}{r^2} - U' = 2g'^2 \frac{Q^2}{r^4}. \tag{6.8}$$

The solution of the Eq. (6.8) can be easy obtained if we use once more the Eq. (6.5). Denoting $A^2 = y$ we obtain the following differential equation for the new unknown function $y(r)$:

$$r^2 y'' - 2y + 2 = 4g'^2 \frac{Q^2}{r^4}. \tag{6.9}$$

Its solution is of the form:

$$y(r) = A^2(r) = 1 + \frac{\alpha}{r} + \frac{Q^2}{r^2} + \beta r^2, \tag{6.10}$$

where α and β are two constants of integration. It is known [16] that the constant α is determined by the mass m of the point-like source that creates the gravitational field by the relation $\alpha = -2m$. The other constant β is determined by the condition that the solution (6.10) verifies the Eqs. (6.7a)–(6.7b) that now coincide as a consequence of the Eq. (6.5). Introducing the solution (6.10) into the Eq. (6.7a), we obtained:

$$\beta = 4\lambda^2 = -\frac{\Lambda}{3}; \quad \Lambda = -12\lambda^2. \tag{6.11}$$

Finally, the solutions of the field equations (6.3) and (6.5) are:

$$A^2(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2; \quad U(r) = -\frac{m}{r^2} + \frac{Q^2}{r^3} + \frac{\Lambda}{3}r. \tag{6.12}$$

Here Λ is the cosmological constant of the model [9]. If we consider the contraction $\lambda \to 0$, then the *DS* group becomes the Poincaré group, and the solutions reduces to the Reissner-Nordström one.

We can define the dual tensor ${}^*A_{\mu\nu}$ of the electromagnetic field by the relation [17]

$$ {}^*A_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{\mu\nu\rho\sigma}A^{\rho\sigma}, \tag{6.13}$$

where $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita symbol of rank four $\varepsilon_{0123} = +1$. We use the spherically symmetric Minkowski metric $g_{\mu\nu}$ in Eq. (2.1), to rise and lower the space-time indices $\mu, \nu, \rho, \sigma = 0, 1, 2, 3$ and denote $g = \det(g_{\mu\nu})$. The condition of self-duality for the electromagnetic field means, in fact, the duality between the electric and magnetic fields. The duality transformation changes the field of an electric charge in the magnetic one (of a magnetic monopole) and the inverse map is also satisfied. This is a well known property (the duality) of electromagnetic field. If we consider $Q = e$ (the electrical charge of the electron) and denote the magnetic charge of the monopole by δ, then the duality imposes the relation:

$$\frac{\delta}{e} = \frac{1}{2\alpha}, \tag{6.14}$$

where $\alpha = \dfrac{e^2}{4\pi\varepsilon_0\hbar c} = \dfrac{1}{137}$ is the constant of fine structure.

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