OMNÈS REPRESENTATIONS WITH INELASTIC EFFECTS FOR HADRONIC FORM FACTORS

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We derive a generalized Omnès representation for the hadronic form factors, which satisfies Watson theorem in the elastic region and includes the effects of inelastic channels. As an application we discuss the behaviour of the scalar form factors of the pion near the $K\bar{K}$ threshold. The results are useful also for the calculation of the phases produced by the strong final state interactions in the nonleptonic decays of the K and B mesons.

1. INTRODUCTION

The hadronic form factors, defined as matrix elements of operators bilinear in the quark fields among hadronic states, play an important role both in perturbative and nonperturbative quantum chromodynamics (QCD). Perturbative QCD predicts the behaviour of the form factors only at large momentum transfer in the space-like region, where asymptotic freedom holds and the hadronic thresholds are absent [1]. On the other hand, at low energies, chiral perturbation theory (ChPT) provides a systematic expansion of these quantities in powers of the momenta and the quark masses. The form factors of the light pseudoscalar mesons were calculated to one-loop in [2] and beyond this order in [3]. The complete evaluation to two loops is given in [4].

Dispersion theory provides also a powerful tool for studying the form factors and relating their low and the high energy behaviours. In the complete theory of QCD, which includes confinement, the form factors are analytic functions of real type in the complex energy plane cut along the real axis from the threshold imposed by unitarity to infinity. The most convenient dispersive representation is the so-called Omnès representation [5], which expresses the form factors in terms of their phase along the cut. This representation allows an easy implementation of Watson theorem [6], which states that in the elastic region the phase of the form factor is equal to the phase of the elastic final state scattering. Many applications of the Omnès representation and its mathematical generalizations for various weak and electromagnetic form factors exists in the literature(see for instance [7] where dispersive and chiral symmetry constraints on the light meson form factors were derived).

The inclusion of the inelastic channels in the Omnès representation is necessary for calculating quantities of interest for ChPT like the quadratic radius of the pion. In particular, the effect of the $K\bar{K}$ channel on the scalar form factors of the pion was recently a controversial subject. The problem was investigated in the frame of a two-channel generalization of the Omnès representation (the so-called Mushkhelishvili-Omnès (M-O) equations [8]) in [9], [10], and more recently in [11]. The conclusion of these works is that the opening of the $K\bar{K}$ channel can have important effects on the phase of the scalar form factors around 1 GeV, the behaviour depending strongly of the quark structure of the corresponding operator. In Ref. [12], on the other hand, it is claimed that the effect of inelasticity is negligible. As this conclusion is based on a single-channel Omnès formalism, it is of interest to include the effect of inelasticity in this formalism. In the present paper we address this problem and write down a single-channel Omnès representation which includes explicitly the influence of the inelastic channels. We stress that the complete solution is provided only by solving the coupled-channels M-O equations. The formulae which we derive are useful however since they offer a rather transparent picture of the inelastic effects, allowing us to understand in a qualitative way the different behaviour of the various form factors. Also, understanding the inelastic channels is crucial for predicting the effect of the final state interactions in nonleptonic weak decays like $K \to \pi\pi$ or $B \to \pi\pi$.

2. DISPERSIVE REPRESENTATION IN TERMS OF THE PHASE

We consider the scalar form factors of the pion

$$\Gamma(s) = \langle \pi(p)\pi(p') \,|\, m_u \bar{u}u + m_d \bar{d}d \,|\, 0 \rangle \tag{1}$$

and

$$\Delta(s) = \langle \pi(p)\pi(p') \,|\, m_s \bar{s}s \,|\, 0 \rangle, \tag{2}$$

where $s = (p + p')^2$, u, d, s are the quark fields and m_u, m_d, m_s their current masses. For simplicity, we denote generically the above form factors by $F(s)$. The function $F(s)$ is analytic in the s-plane cut from the elastic threshold at $s = 4m_\pi^2$ to infinity. The phase $\delta_F(s)$ of $F(s)$ on the cut is defined by the boundary condition

$$F(s + i\epsilon) = \mathrm{e}^{2i\delta_F(s)} F(s - i\epsilon), \quad s > 4m_\pi^2. \tag{3}$$

This relation represents a Riemann boundary value problem [8], with the general solution [5]:

$$F(s) = P_n(s)\exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_F(s')\mathrm{d}s'}{s'(s'-s)}\right], \tag{4}$$

where $P_n(s)$ is a polynomial of degree n.

Perturbative QCD predicts the asymptotic behaviour [1]

$$F(s) \sim \frac{\alpha_s(-s)}{-s}, \quad |s| \to \infty, \tag{5}$$

where $\alpha_s(-s) = 4\pi/9\ln(-s/\Lambda^2)$ is the QCD running coupling. From the Omnès representation (4), this implies the asymptotic behaviour of the phase

$$\delta_F(s) \sim (n+1)\pi + \frac{\pi}{\ln\frac{s}{\Lambda^2}}, \quad s \to \infty. \tag{6}$$

Chiral expansions suggest that the form factor $\Gamma(s)$ has no zeros in the complex plane, which means that $n = 0$, the polynomial in (4) reduces to a constant and

$$\delta_\Gamma(s) \sim \pi + \frac{\pi}{\ln\frac{s}{\Lambda^2}}, \quad s \to \infty. \tag{7}$$

On the other hand, for the form factor $\Delta(s)$, ChPT indicates a zero close to $s = 0$, which means that in this case $n = 1$ and the asymptotic behaviour of the phase is

$$\delta_\Delta(s) \sim 2\pi + \frac{\pi}{\ln\frac{s}{\Lambda^2}}, \quad s \to \infty. \tag{8}$$

3. UNITARITY RELATION AND WATSON THEOREM

Along the cut $s > 4m_\pi^2$, the form factor $F(s)$ satisfies the unitarity relation

$$\mathrm{Im}F(s) = \sigma(s)F(s)[f_0^{(0)}(s)]^* + \sigma_{in}(s), \tag{9}$$

where $\sigma(s) = \sqrt{1-4m_\pi^2/s}$ and the isoscalar S-partial wave $f_0^{(0)}(s)$ is parametrized as

$$f_0^{(0)}(s) = \frac{\eta_0^{(0)}e^{2i\delta_0^{(0)}}-1}{2i\sigma(s)}, \tag{10}$$

in terms of the elasticity $\eta_0^{(0)} \le 1$ and the phase-shift $\delta_0^{(0)}$. All the functions in (9) are evaluated on the upper edge of the cut. Neglecting the small contribution of four pions, the inelastic term $\sigma_{in}(s)$ can be approximated at low energies by the contribution of the $K\overline{K}$ channel

$$\sigma_{in}(s) = \theta(s - 4m_K^2)\sigma_K(s)F_K(s)T_{\pi K}^*(s), \tag{11}$$

where $\sigma_K = \sqrt{1 - 4m_K^2/s}$ is the phase space, $F_K(s)$ is the kaon scalar form factor defined by replacing in Eqs. (1) or (2) the pion pair by a $K\overline{K}$ pair, and $T_{\pi K}$ denotes the $\pi\pi \to K\overline{K}$ S-wave amplitude.

Eq. (9) can be written as

$$F(s + i\epsilon)\left(1 - 2i\,\sigma(s)\left[f_0^{(0)}(s)\right]^*\right) - F(s - i\epsilon) = 2i\,\sigma_{in}(s), \tag{12}$$

or, using (10), as:

$$F(s + i\epsilon)\eta_0^{(0)}(s)\mathrm{e}^{-2i\delta_0^{(0)}(s)} - F(s - i\epsilon) = 2i\,\sigma_{in}(s). \tag{13}$$

In the elastic region $s < 4m_K^2$, where $\eta_0^{(0)} = 1$ and $\sigma_{in}(s) = 0$, Eq. (13) reduces to

$$F(s + i\epsilon) = \mathrm{e}^{2i\delta_0^{(0)}(s)}F(s - i\epsilon), \quad s < 4m_K^2, \tag{14}$$

which, compared to (3), yields Watson theorem $\delta_F(s) = \delta_0^{(0)}(s)$ for $s < 4m_K^2$. Above the $K\overline{K}$ threshold the phase δ_F is no longer equal to the phase shift.

4. OMNÈS REPRESENTATIONS WITH INELASTICITY

Equation (13) has the form of an nonhomogeneous Riemann boundary value problem, whose general solution is given in [8]. Following [16], we look for solutions F which satisfy elastic unitarity and time reversal invariance, which requires that the form factor is real-analytic in the cut plane, $F(s^*) = F^*(s)$. We look for solutions of the form

$$F(s) = G(s)O(s), \tag{15}$$

where $O(s)$ is an Omnès function defined in terms of a certain phase, and $G(s)$ a residual function which accounts for the inelasticity. To satisfy Watson theorem, the phase of $O(s)$ must be equal to $\delta_0^{(0)}(s)$ below the inelastic threshold, but above it the phase is arbitrary. For every choice of the phase of $O(s)$ we calculate the remaining function G from the unitarity relation.

4.1. OMNÈS FUNCTION DEFINED WITH THE PHASE SHIFT

A natural choice is to take the phase of O equal to the phase shift $\delta_0^{(0)}$ along the whole cut up to infinity. So we write the form factor as

$$F(s) = G_1(s)O_1(s) \tag{16}$$

where

$$O_1(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}s' \frac{\delta_0^{(0)}(s')}{s'(s'-s)}\right]. \tag{17}$$

In order to calculate the residual function G_1 appearing in (16), we first notice that by Watson theorem it is real below the inelastic threshold, so it is analytic in the s-plane cut only for $s > 4m_K^2$. From (16) we have

$$\begin{aligned}\mathrm{Re}\, G_1(s) &= \frac{1}{|O_1(s)|}[\mathrm{Re}F\cos\delta_0^{(0)} + \mathrm{Im}F\sin\delta_0^{(0)}],\\ \mathrm{Im}\, G_1(s) &= \frac{1}{|O_1(s)|}[\mathrm{Im}F\cos\delta_0^{(0)} - \mathrm{Re}F\sin\delta_0^{(0)}].\end{aligned} \tag{18}$$

On the other hand, by multiplying both sides of (13) with $\mathrm{e}^{i\delta_0^{(0)}(s)}$ and taking the real and imaginary part we have

$$\begin{aligned}\mathrm{Re}F\cos\delta_0^{(0)} + \mathrm{Im}F\sin\delta_0^{(0)} &= \frac{2}{1-\eta_0^{(0)}}\mathrm{Im}\left[\sigma_{in}\mathrm{e}^{i\delta_0^{(0)}}\right]\\ \mathrm{Im}F\cos\delta_0^{(0)} - \mathrm{Re}F\sin\delta_0^{(0)} &= \frac{2}{1+\eta_0^{(0)}}\mathrm{Re}\left[\sigma_{in}\mathrm{e}^{i\delta_0^{(0)}}\right].\end{aligned} \tag{19}$$

By comparing with (18) we have

$$\begin{aligned}\mathrm{Re}\, G_1(s) &= \frac{2}{1-\eta_0^{(0)}}\frac{\mathrm{Im}[\sigma_{in}\,\mathrm{e}^{i\delta_0^{(0)}}]}{|O_1(s)|},\\ \mathrm{Im}\, G_1(s) &= \frac{2}{1+\eta_0^{(0)}}\frac{\mathrm{Re}[\sigma_{in}\,\mathrm{e}^{i\delta_0^{(0)}}]}{|O_1(s)|}.\end{aligned} \tag{20}$$

Therefore, the function $G_1(s)$ in the representation (16) satisfies (up to a polynomial real on the cut) the Omnès representation

$$G_1(s) = \exp\left[\frac{s}{\pi}\int_{4m_K^2}^{\infty} \mathrm{d}s' \frac{\psi_1(s')}{s'(s'-s)}\right] \tag{21}$$

where $\psi_1(s)$ is the argument of $\mathrm{Re}G_1(s)+i\mathrm{Im}G_1(s)$:

$$\psi_1(s)=\mathrm{Arctg}\left[\frac{1-\eta_0^{(0)}}{1+\eta_0^{(0)}}\frac{\mathrm{Re}\left\{\sigma_{in}\,\mathrm{e}^{i\delta_0^{(0)}}\right\}}{\mathrm{Im}\left\{\sigma_{in}\,\mathrm{e}^{i\delta_0^{(0)}}\right\}}\right]. \tag{22}$$

From (16), (17) and (21) it follows that the phase δ_F of the form factor is given by

$$\delta_F(s)=\delta_0^{(0)}+\psi_1(s). \tag{23}$$

4.2. OMNÈS FUNCTION DEFINED WITH THE PHASE OF THE PARTIAL WAVE AMPLITUDE

An alternative choice is to take the phase of the Omnès function O in (15) equal to the phase δ_t of the partial wave amplitude, defined by

$$\delta_t(s)=\mathrm{Arctg}\left[\frac{\mathrm{Im}f_0^{(0)}(s)}{\mathrm{Re}f_0^{(0)}(s)}\right]=\frac{1-\eta_0^{(0)}\cos 2\delta_0^{(0)}}{\eta_0^{(0)}\sin 2\delta_0^{(0)}}. \tag{24}$$

So, we take

$$F(s)=G_2(s)O_2(s) \tag{25}$$

where

$$O_2(s)=\exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\mathrm{d}s'\frac{\delta_t(s')}{s'(s'-s)}\right]. \tag{26}$$

In the elastic region $\delta_t=\delta_0^{(0)}$, but above the $K\bar{K}$ threshold this equality is no longer valid[1]. In fact, the phase-shift $\delta_0^{(0)}$ has the peculiarity that it raises rapidly near the $K\bar{K}$ threshold reaching the value π above (and close to) it. From Eq. (24) it folows that when $\eta_0^{(0)}<1$ and $\delta_0^{(0)}$ passes through π, the phase δ_t has a dip, which becomes steeper when the elasticity is close to 1. So, above the inelastic threshold the phase δ_t of the amplitude is quite different from the phase shift $\delta_0^{(0)}$.

[1] Actually, if $\delta_0^{(0)}(s_0)=\pi$ for $s_0<4m_K^2$, then at this point phase δ_t makes a jump by $-\pi$, $\delta_0^{(0)}(s_0)-\delta_t(s_0)=\pi$, in order to preserve the positivity of the imaginary part $\mathrm{Im}\,f_0^{(0)}=\sin^2\delta_0^{(0)}$. We shall assume that $\delta_0^{(0)}$ reaches π only above the $K\bar{K}$ threshold, as indicate most experimental parametrizations, so that $\delta=\delta_0^{(0)}$ in the whole elastic region.

In order to calculate the residual function G_2 defined in (25) we notice that for $s > 4m_K^2$ the two terms in the r.h.s. of Eq. (9) are not separately real, but their sum must be real. Taking the real and the imaginary parts of (9) we have:

$$\begin{aligned} \mathrm{Re}\,\sigma_{in} &= \mathrm{Im}F - \mathrm{Re}\left\{\sigma F[f_0^{(0)}]^*\right\}, \\ \mathrm{Im}\,\sigma_{in} &= -\mathrm{Im}\left\{\sigma F(s)[f_0^{(0)}]^*\right\}. \end{aligned} \tag{27}$$

Inserting in these relations the representation (25) and noting that with the choice (26) the product $O_2[f_0^{(0)}]^* = |O_2||f_0^{(0)}|$ is real, we obtain

$$\begin{aligned} \mathrm{Im}G_2(s) &= -\frac{\mathrm{Im}\,\sigma_{in}}{\sigma|O_2(s)||f_0^{(0)}|} \\ \mathrm{Re}G_2(s) &= \frac{\mathrm{Re}\,\sigma_{in}/|O_2| - \mathrm{Im}\,G_2(s)\cos\delta_t}{\sin\delta_t - \sigma|f_0^{(0)}|}. \end{aligned} \tag{28}$$

Denoting by $\psi_2(s)$ the argument of $\mathrm{Re}G_2(s) + i\,\mathrm{Im}G_2(s)$

$$\psi_2(s) = \mathrm{Arctg}\left[\frac{\mathrm{Im}\,G_2(s)}{\mathrm{Re}\,G_2(s)}\right], \tag{29}$$

we write the function G_2, up to a polynomial, as

$$G_2(s) = \exp\left[\frac{s}{\pi}\int_{4m_K^2}^{\infty} \mathrm{d}s' \frac{\psi_2(s')}{s'(s'-s)}\right] \tag{30}$$

From (25), (26) and (30) it follows that the phase δ_F of the form factor is given by

$$\delta_F(s) = \delta_t + \psi_2(s). \tag{31}$$

5. COMMENTS ON THE SCALAR FORM FACTORS OF THE PION

The two approaches described above are equivalent and must lead to identical results. We checked this equivalence fot the numerical solutions of the two-channel M–O equations calculated in [11] using the experimental data from [13].

More exactly, we evaluated the right hand sides of the relations (23) and (31) using as input the corresponding quantities calculated in [11] and checked that they lead to identical results, which moreover coincide with the phase of the form factor obtained in [11]. The results are shown in Fig. 1 (reproduced from [11]), where we indicate the phase shift $\delta_0^{(0)}$, the phase δ_t of the partial wave, and the phases of the form factors Γ and Δ. Below the opening of the inelastic channels all the phases depicted are equal. Above the $K\bar{K}$ threshold, δ_Γ has a pronounced dip and then follows closely the phase δ_t of the scattering amplitude, staying below the phase shift $\delta_0^{(0)}$ by approximately $-\pi$. This behaviour of the form factor Γ is confirmed by the experimental data on the central production of pion pairs in *pp* collisions [17]. In the notations of the previous section, this means that for the form factor Γ the additional phase ψ_1 from (23) is negative and approaches rapidly the value $-\pi$, while the additional phase ψ_2 from (31) is close to zero. On the other hand, the phase of the form factor Δ follows closely the phase shift also above the $K\bar{K}$ threshold, which means that the additional phase ψ_1 from (23) is small, while the phase ψ_2 from (31) is large and close to π.

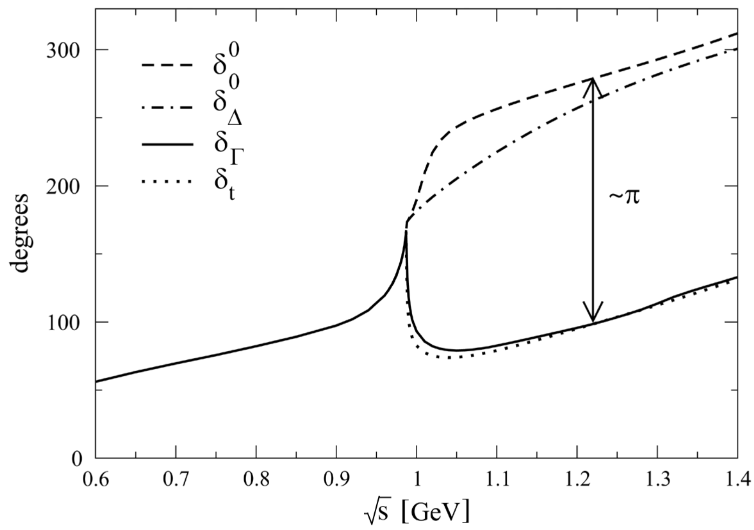

Fig. 1. – The phase δ_Γ of the pion form factor $\Gamma(s)$ calculated from the two-channel M-O equations [11] (solid line). The dashed and dotted lines describe the phase shift $\delta_0^{(0)}$ and the phase δ_t of the partial wave amplitude, respectively. The dash-dotted line depicts the phase δ_Δ of the form factor $\Delta(s)$.

This behaviour, obtained numerically in [11], can be understood qualitatively using the expressions derived in Section 4. For illustration, we consider the representation given in subsection 4.1, where the Omnès function is expressed in terms of the phase shift. As we mentioned, the experimental data on $\pi\pi$ scattering [13] indicate that the phase shift $\delta_0^{(0)}(s)$ raises very rapidly and reaches the value π just above the inelastic $K\bar{K}$ threshold. Moreover, just above this threshold the elasticity $\eta_0^{(0)}$ has a sharp decrease, indicating a very strong inelasticity, and then rather quickly approaches again the elastic value $\eta_0^{(0)} = 1$.

From the expression (22) it follows that, if the elasticity $\eta_0^{(0)}(s)$ is close to 1, the additional phase ψ_1 appearing in (23) is equal to 0 *modulo* ±π. Denoting by $\delta_{\sigma_{in}}$ the phase of the complex quantity $\sigma_{in} = |\sigma_{in}| \exp(i\delta_{\sigma_{in}})$ and omitting the irrelevant positive factors, we notice from (18) that the phase ψ_1 depends on the signs of the quantities

$$\mathrm{Re}\, G_1 \sim \sin(\delta_{\sigma_{in}} + \delta_0^{(0)})/(1 - \eta_0^{(0)}) \qquad (32)$$

$$\mathrm{Im}\, G_1 \sim \cos(\delta_{\sigma_{in}} + \delta_0^{(0)}). \qquad (33)$$

In the vicinity of the threshold the phase $\delta_{\sigma_{in}}$ is suppressed by the phase space. Since, as we mentioned above, the phase shift $\delta_0^{(0)}$ is close to π, the quantity $\cos(\delta_{\sigma_{in}} + \delta_0^{(0)})$, which determines the sign of the imaginary part of G_1, is negative. Moreover, just above the threshold, where $\delta_0^{(0)}$ is still less than π, the real part $\mathrm{Re}\, G_1$ defined in (32) is positive. This means that the point associated to the complex quantity G_1 is situated in the fourth quadrant of the trigonometric circle, and $\psi_1 < 0$. From (23) it follows therefore that the inelasticity has the effect of lowering the phase of the form factor. The evolution of the phase ψ_1 at higher energies depends on the sign of the real part $\mathrm{Re}\, G_1$. If

$$\sin(\delta_{\sigma_{in}} + \delta_0^{(0)}) > 0 \qquad (34)$$

then $\mathrm{Re}\, G_1 > 0$ and the point associated to G_1 remains in the fourth quadrant. Hence, when the inelasticity $\eta_0^{(0)}$ approaches again the value 1, the phase ψ_1 tends to 0 through negative values, and the phase of the form factor approaches $\delta_0^{(0)}$ from below. But if

$$\sin(\delta_{\sigma_{in}} + \delta_0^{(0)}) < 0, \qquad (35)$$

then the point associated to G_1 enters the third quadrant, and $\psi_1 \to -\pi$ when $\eta_0^{(0)}$ becomes close to 1. The decisive role is played therefore by the phase of the inelastic term $\sigma_{in} \sim F_K(s) T_{\pi K}^*(s)$ defined in (11). This quantity can be understood by noticing that the two-channel unitarity equations given in [9] are satisfied by the following expressions:

$$\begin{aligned} F(s) &= c_1(s) T_{\pi\pi}(s) + c_2(s) T_{\pi K}(s) \\ F_K(s) &= c_1(s) T_{\pi K}(s) + c_2(s) T_{KK}(s), \end{aligned} \qquad (36)$$

where $T_{\pi\pi} = f_0^{(0)}$, $T_{\pi K}$ and T_{KK} denote the S-wave projections of the $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$ and $K\bar{K} \to K\bar{K}$ amplitudes, respectively, and the functions $c_1(s)$ and $c_2(s)$ are real for $s > 4m_\pi^2$. If the coefficients c_1 and c_2 are positive, the relations (36) imply, by the parallelogram rule for vector addition, that the phase of the pion form factor F_π is larger than the phase of $T_{\pi\pi}$ and smaller than the phase of $T_{\pi K}$, while the phase of the kaon form factor F_K is larger than the phase of T_{KK} and smaller than the phase of $T_{\pi K}$. We recall that by unitarity the phase of the nondiagonal amplitude $T_{\pi K}$ is the sum of the phase shifts of the diagonal elements [9]. The experimental data [13]–[15] indicate that the phase shift δ_K of the $K\bar{K} \to$ $\to K\bar{K}$ transition is negative. Using a relation similar to (10) (with $\delta_0^{(0)}$ replaced by δ_K), we obtain for the phase of T_{KK} positive values in the second quadrant.

Let us consider first the form factor $\Gamma(s)$ defined in (1). The coefficients c_1 and c_2 take values consistent with the asymptotic condition (7). The explicit calculation with data from [13] indicate that $\Gamma(s) \sim c_1(s) T_{\pi\pi}(s)$, which implies that $\delta_\Gamma \sim \delta_t$. The Omnès formalism is consistent with this result: inserting $\Gamma_K(s) \sim c_1(s) T_{\pi K}(s)$ in the expression (11), it follows that the $\delta_{\sigma_{in}}$ is close to 0 and the relevant quantity in Eq. (32) is $\sin(\delta_{\sigma_{in}} + \delta_0^{(0)}) \sim \sin\delta_0^{(0)}$. Since above the $K\bar{K}$ threshold $\delta_0^{(0)}$ becomes rapidly greater than π, the inequality (35) holds, which means that the difference $\delta_\Gamma - \delta_0^{(0)}$ tends to $-\pi$, as shown in Fig. 1. This result is quite stable with respect to the parametrizations of the unitary S-matrix. Indeed, even if the first term in (36) is not dominant and the phase of σ_{in} is negative, the correction is not very large, and still leads to $\sin(\delta_{\sigma_{in}} + \delta_0^{(0)}) < 0$, due to the large values of $\delta_0^{(0)}$.

In the case of the form factor Δ defined in (2), the asymptotic condition (8) select a different pattern for the coefficients c_1 and c_2, which may be negative. It follows that $\delta_{\sigma_{in}}$ is a large negative phase, so that the sum $\delta_{\sigma_{in}} + \delta_0^{(0)}$ becomes less than π and the inequality (34) holds. Therefore, when the elasticity $\eta_0^{(0)}$ approaches 1, the difference $\delta_\Delta - \delta_0^{(0)}$ tends to 0.

6. CONCLUSIONS

In the present paper we derived single channel Omnès representations for the hadronic form factors, which include explicitly the effects of the inelastic channels in the unitarity sum. As we discussed in Section 4, the results provide a qualitative understanding of the scalar form factors of the pion in the vicinity of the inelastic $K\bar{K}$ threshold. We mention that the results are useful also for including the effects of final state rescattering in the nonleptonic decays like $K \to \pi\pi$ and $B \to \pi\pi$, which are of interest for the CP-violation parameters in the Standard Model. Dispersion relations and Omnès representations for the amplitudes of these decays, including the effects of initial and final state interactions, were derived in [18], [19]. The function G_1 describing the inelastic channels in $K \to \pi\pi$ decay was expanded in a power series based on a conformal mapping [19]. In the case of B nonleptonic decays, the additional phase (22) produced by the final state interactions can be evaluated in terms of the weak decay amplitudes into intermediate pseudoscalar and vector mesons, using Regge theory for the strong rescattering amplitudes [20].

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