PLANE SYMMETRIC COSMOLOGICAL MODEL OF INTERACTING FIELDS IN GENERAL RELATIVITY

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We have investigated the plane symmetric models of interacting field. To get a solution, here we consider two cases a stiff fluid or Zeldovich fluid and disordered radiation. Also, some physical and geometric properties of the models are discussed.

Key words: Plane symmetry, cosmology, interacting fields, stiff fluid or Zel'dovich fluid, disordered radiation.

1. INTRODUCTION

The plane symmetric cosmological model plays an important role in the presence of linearly coupled massless scalar field and source of free electromagnetic field and perfect fluid. A stiff fluid and disordered radiation equation of state is of much interest in general theory of relativity because it represents the most extreme conceivable case for a perfect fluid. In this equation the speed of light is equal to the speed of sound. Mohanty *et al.* [1, 2] have investigated the charged stiff perfect fluid in presence of massless scalar fields with cylindrically symmetric metric in comoving coordinate system. Mohanty [3] have discussed the problem of interacting fields using relativistic cylindrically symmetric metric. Panigrahi and Sahu [4] have studied a micro and macro cosmological models in presence of massless scalar field with perfect fluid.

Singh and Abdussattar [5]; Roy and Bali [6] obtained a static spherically and cylindrically symmetric solutions to Einstein field equations for perfect fluid filled with disordered radiation. Teixeira, Wolk, Som [7] investigated a model filled with source free disordered distribution of electromagnetic radiation in

General Relativity. Non-Static plane symmetric cosmological model filled with disordered radiation obtained by Roy and Singh [8]. Magnetic field has an important role in the study of the cosmological model. The importance of the magnetic field for various astronomical phenomena has been studied by Tikekar and Patel [9], Thorne [10], Collins [11], Roy and Prakash [12], Dunn and Tupper [13]. Recently Pawar *et al.* [14] studied the magnetized plane symmetric viscous fluid cosmological model in General theory of relativity.

In this paper, we shall discussed the plane symmetric cosmological model in the presence of linearly coupled massless scalar field and source free electromagnetic field and stiff perfect fluid. This paper is organized as follows. The metric and field equations are presented in section 2. In section 3, we deal with solutions of field equation in two cases for stiff fluid model and disordered radiation model with their physical and kinematical properties. In section 4, we have given the concluding remarks.

2. THE METRIC AND FIELD EQUATIONS

Consider plane symmetric metric in the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 , \tag{1}$$

where A and B are functions of t only.

The relativistic field equations for linearly coupled charged perfect fluid and massless scalar fields are

$$G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R = -8\pi(E_{ij} + T_{ij} + S_{ij}), \tag{2}$$

where G_{ij} is the Einstein's tensor, R_{ij} is the Ricci tensor, R is the Ricci scalar, E_{ij} is the stress energy momentum tensor of electromagnetic field, T_{ij} is energy momentum tensor of massless scalar field and S_{ij} is the energy momentum tensor of perfect fluid distribution respectively. Here the units are chosen so that the velocity of light c = 1 and gravitational constant G = 1.

The electromagnetic energy momentum tensor is

$$E_{ij} = \frac{1}{4\pi}\left[F_{i\alpha} F_j^{\alpha} - \frac{1}{4} g_{ij} F_{\alpha\beta} F^{\alpha\beta} \right], \tag{3}$$

where F_{ij} is the electromagnetic field tensor derived form the four potential ϕ_i and is defined as

$$F_{ij} = \varphi_{i,j} - \varphi_{j,i} , \tag{4}$$

$$F^{ij}_{;j} = -4\pi\sigma U^i . \tag{5}$$

In comoving coordinate system, the magnetic field is taken along z-axis, so that the only non-vanishing component of electromagnetic field tensor F_{ij} is F_{12}.

The first set of Maxwell's equation

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0. \tag{6}$$

Leads to

F_{12} = constant = H, other all components are zero.

The energy momentum tensor T_{ij} for massless scalar field is given by

$$T_{ij} = V_{,i}V_{,j} - \frac{1}{2}g_{ij}V_{,s}V^{,s} \tag{7}$$

and massless scalar field V also satisfy the equation

$$g^{ij}V_{;ij} = \sigma, \tag{8}$$

where σ is the charge density, comma (,) and semicolon (;) denotes partial and covariant differentiation respectively.

Also, the energy momentum tensor S_{ij} for perfect fluid distribution is given by

$$S_{ij} = (p+\rho)U_iU_j + g_{ij}p, \tag{9}$$

together with

$$g^{ij}U_iU_j = -1, \tag{10}$$

where p, ρ and U^i are internal pressure, rest mass density and four velocity vectors of the distribution respectively.

From (3), (7) and (9), the non vanishing components of E_{ij}, T_{ij} and S_{ij}, respectively are

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{H^2}{8\pi A^4}, \tag{11}$$

$$T_1^1 = T_2^2 = T_3^3 = -T_4^4 = \frac{-\dot{V}^2}{2}, \tag{12}$$

and $$S_1^1 = S_2^2 = S_3^3 = -p, \quad S_4^4 = \rho. \tag{13}$$

With the help of equation (11) to (13), the field equation (2) for the metric (1) can be written as

$$\left(\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B}\right) = 8\pi\left(\frac{H^2}{8\pi A^4} - p - \frac{1}{2}\dot{V}^2\right), \tag{14}$$

$$\left(\frac{\dot{A}^2}{A^2}+2\frac{\ddot{A}}{A}\right)=8\pi\left(\frac{-H^2}{8\pi A^4}-p-\frac{1}{2}\dot{V}^2\right), \tag{15}$$

$$\left(\frac{\dot{A}^2}{A^2}+\frac{2\dot{A}\dot{B}}{AB}\right)=8\pi\left(\frac{-H^2}{8\pi A^4}+\rho+\frac{1}{2}\dot{V}^2\right). \tag{16}$$

Here dot denote differentiation w. r. t. ‘t’.

We shall determine the exact solution of field equations using following two cases.

3. SOLUTIONS

CASE I: ZELDOVICH FLUID OR STIFF FLUID

The set of field equations (14)–(16) contains three equations with five unknowns A, B, p, ρ and V. An additional constraint relating these parameters is required to obtained explicit solutions of the system.

We assume a relation between the metric potential:

$$A = B^n. \tag{17}$$

We have,

$$p = \rho. \tag{18}$$

Using (17), (18) the set of field equation (14)–(16) reduce to

$$B^{(4n+1)}\ddot{B}+\alpha B^{4n}\dot{B}^2=\frac{-2H^2B^2}{(3n+1)}. \tag{19}$$

This can be written as

$$\frac{df^2}{dB}+2\frac{\alpha}{B}f^2=\frac{-4H^2B^2}{(3n+1)B^{(4n+1)}},$$

where

$$\alpha = 2n \tag{20}$$

and

$$\dot{B}=f(B). \tag{21}$$

From (21), we obtain

$$\left(\frac{dB}{dt}\right)^2=\left(\frac{-2H^2}{(3n+1)B^{(4n-2)}}+\frac{C}{B^{4n}}\right), \tag{22}$$

where C is the integration constant.

Using F_{12} = constant = (H) and $U^4 \neq 0$, the equation (3) gives the values of charge density σ as

$$\sigma = 0. \tag{23}$$

Using equation (23), the equation (8) can be written as

$$\ddot{V} + \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{V} = 0. \tag{24}$$

After solving this equation, we have

$$\dot{V} = \frac{C_1}{B^{(2n+1)}}. \tag{25}$$

After suitable transformation of coordinates, the metric (1) reduces to the form

$$ds^2 = \left(\frac{-2H^2}{(3n+1)T^{4n-2}} + \frac{C}{T^{4n}}\right)^{-\frac{1}{2}} dT^2 - T^{2n}(dx^2 + dy^2) - T^2 dz^2. \tag{26}$$

3.1. SOME PHYSICAL AND KINEMATICAL PROPERTIES OF THE MODEL

Some physical and kinematical properties of the plane symmetric cosmological model (26) obtained in presence of linearly coupled massless scalar field and source free electromagnetic field with stiff perfect fluid.

The pressure and density of model (26) are

$$8\pi p = 8\pi\rho = \frac{(n^2 + 2n)C - 4\pi C^2}{T^{4n+2}} - \frac{(2n^2 + 3n + 1)H^2}{(3n+1)T^{4n}}. \tag{27}$$

Spatial Volume $$V = \sqrt{-g} = T^{(2n+1)}. \tag{28}$$

As $T \to 0$, spatial volume $V \to 0$ as $T \to \infty$, spatial volume $V \to \infty$ which shows that the universe starts expanding with zero volume and blows up at infinite past and future.

Expansion scalar $$\theta = \frac{1}{3}U^i_{;i} = \frac{(2n+1)}{3T}\left[\frac{-2H^2}{(3n+1)T^{4n-2}} + \frac{C}{T^{4n}}\right]^{\frac{1}{2}}. \tag{29}$$

From equation (29) we can observe that $\theta \to 0$ as $T \to \infty$ and $\theta \to \infty$ as $T \to 0$. Thus universe is expanding with increases in time but rate of expansion becomes slow as time increases.

Shear scalar $$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{(2n+1)^2}{54T^2}\left[\frac{-2H^2}{(3n+1)T^{4n-2}} + \frac{C}{T^{4n}}\right]. \tag{30}$$

From equation (30), we observe that $\sigma^2 \to 0$ as $T \to \infty$ and $\sigma^2 \to \infty$ as $T \to 0$. Thus the shape of the universe changes uniformly.

CASE II: DISORDERED RADIATION

Here, we assume a relation between the metric potential:

$$A = B^n. \tag{31}$$

We have,

$$\rho = 3p. \tag{32}$$

Using (31), (32) and (24) the set of field equation (14) to (16) reduce to

$$\ddot{B} + \beta \frac{\dot{B}^2}{B} = \frac{-8\pi c_1^2}{(4n+2)B^{(4n+1)}}. \tag{33}$$

This can be written as

$$\frac{df^2}{dB} + 2\frac{\beta}{B} f^2 = \frac{-16\pi c_1^2}{(4n+2)B^{(4n+1)}},$$

where

$$\beta = \frac{6n^2}{(4n+2)} \tag{34}$$

and

$$\dot{B} = f(B). \tag{35}$$

From (35), we obtain

$$\left(\frac{dB}{dt}\right)^2 = \left(\frac{4\pi c_1^2}{(n^2+2n)B^{\frac{16n^2+8n}{4n+2}}} + \frac{c_2}{B^{\frac{6n^2}{2n+1}}} \right). \tag{36}$$

where c_1 and c_2 are the integration constants.
After suitable transformation of coordinates, the metric (1) reduces to the form

$$ds^2 = \left(\frac{4\pi c_1^2}{(n^2+2n)T^{\frac{16n^2+8n}{4n+2}}} + \frac{c_2}{T^{\frac{6n^2}{2n+1}}} \right)^{-1} dT^2 - T^{2n}(dx^2+dy^2) - T^2 dz^2. \tag{37}$$

3.2. SOME PHYSICAL AND KINEMATICAL PROPERTIES OF THE MODEL

Some physical and kinematical properties of the plane symmetric cosmological model (37) are obtained as follows.

The pressure and density are given by

$$8\pi p = \frac{(2n+3)4\pi c_1^2}{(1+2n)T^{\frac{8n^2+4n+6}{2n+1}}} + \frac{(6n-3n^2)c_2}{(1+2n)T^{\frac{6n^2+4n+2}{2n+1}}} - \frac{H^2}{T^{4n}} - \frac{4\pi c_1^2}{T^{4n+2}}. \tag{38}$$

$$8\pi\rho = 3\left[\frac{(2n+3)4\pi c_1{}^2}{(1+2n)T^{\frac{8n^2+4n+6}{2n+1}}} + \frac{(6n-3n^2)c_2}{(1+2n)T^{\frac{6n^2+4n+2}{2n+1}}} - \frac{H^2}{T^{4n}} - \frac{4\pi c_1{}^2}{T^{4n+2}}\right].$$

Spatial Volume $$V = \sqrt{-g} = T^{(2n+1)}. \tag{39}$$

As $T \to 0$, spatial volume $V \to 0$ as $T \to \infty$, spatial volume $V \to \infty$ which shows that the universe starts expanding with zero volume and blows up at infinite past and future.

Expansion scalar

$$\theta = \frac{1}{3}U^i_{;i} = \frac{(2n+1)}{3}\left[\frac{4\pi c_1{}^2}{(n^2+2n)T^{\frac{8n^2+4n+6}{2n+1}}} + \frac{c_2}{(1+2n)T^{\frac{6n^2+4n+2}{2n+1}}}\right]^{\frac{1}{2}}. \tag{40}$$

From equation (40), we can observe that $\theta \to 0$ as $T \to \infty$ and $\theta \to \infty$ as $T \to 0$. Thus universe is expanding with increases in time but rate of expansion becomes slow as time increases.

Shear scalar

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{(2n+1)^2}{54}\left[\frac{4\pi c_1{}^2}{(n^2+2n)T^{\frac{8n^2+4n+6}{2n+1}}} + \frac{c_2}{(1+2n)T^{\frac{6n^2+4n+2}{2n+1}}}\right]. \tag{41}$$

From equation (41), we observe that $\sigma^2 \to 0$ as $T \to \infty$ and $\sigma^2 \to \infty$ as $T \to 0$. Thus the shape of the universe changes uniformly.

Also $\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$, shows that both the model does not approach the isotropy for large values of T.

Rotation tensor $$\omega_{ij} = 0. \tag{42}$$

Rotation $$\omega^2 = \frac{1}{2}\omega_{ij}\,\omega^{ij} = 0. \tag{43}$$

From equations (42) and (43) both the models do not admit rotation and acceleration.

Again, from equations (27), (38) and (39), it is observed that when $T \to 0$, the quantities p and ρ are undetermined but $T \to \infty$ we get $p = \rho = 0$.

4. CONCLUSIONS

We have obtained plane symmetric cosmological model in the presence of linearly coupled massless scalar field and source free electromagnetic field with stiff or Zel'dovich fluid $p = \rho$ and disordered radiation $\rho = 3p$. It is interesting to note that the models turn out to be identical at $T \to 0$ and $T \to \infty$. Generally models are expanding, shearing and non-rotating. We observe that they do not approach isotropy for large value of time T.

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