THE SELF-ORGANIZING UNIVERSE

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Received December 6, 2006

The most recently completed redshift surveys, such as 2dFGRS, reveal spectacularly complex structures in galaxy distribution. These structures are described in terms of filaments, clusters and voids and are usually characterized by using fractal geometry language. In this paper it is shown that the fractal dimension of the large scale distribution of galaxies presents scaling behavior well described by a Verhulst-type law. This result is in agreement with the idea that the Universe we observe today is a self-structured system which emerges from a nonlinear self-organizing phenomenon.

Key words: large scale structure, cosmology, structures formation, nonlinear phenomena.

1. INTRODUCTION

An outstanding problem in modern cosmology is to understand how the formation of large scale structures, such as galaxies and clusters of galaxies can be described within the framework of the standard cosmology. The term "large scale structure" is referred here to a distribution of galaxies on the scales roughly from 40 Mpc to 200 Mpc.

Until the middle of 1970s, it was widely accepted the idea that the galaxies are almost uniformly distributed, with only a small percent in clusters. But in the late 1970's, the improvements in spectrographs technology allowed redshift measurements for a large numbers of galaxies. These early redshift surveys showed huge voids (as large as 100 Mpc) in galaxy distribution, surrounded by "walls" and spanned by "filaments" [1].

The next 25 years have seen an explosive evolution of our understanding of the motion of galaxies and their distribution. A complete review of these years [2] shows how the field has moved from a morphological, qualitative description of this distribution to a rigorous description which allows us to approach the cosmological implications of the observed large scale structure.

To characterize these structures a number of statistical tools have been developed [3–7]. The most widely used approach is the correlation function method. The two-point correlation function represents the probability, in excess from homogeneity, of finding a galaxy at a fixed distance from a random neighbor. The Fourier transform of the correlation function is the power spectrum. Another method is CIC (counts-in-cells): the distribution of the number of galaxies found in cells of a given edge which one lays down atop the size of the distribution. For purely geometrical characterization it is used the box-counting method. It provides in a simple manner the fractal dimension of a distribution and because of its simplicity it will be used in this paper.

The study of large scale structure has opened some major questions:

– What is the topology of the galaxy distribution on various scales? The cosmological principle states that the universe is homogeneous and isotropic on the largest scale; is this in agreement with the observational cosmology?
– How did the galaxy structures form? How might we constrain the expanding universe and models for structure formation with the observations?
– How does the inhomogeneous structure relate to the early evolution of the universe?

The current theory [8] predicts that the large scale structures were born from the smaller structures, galaxies, through a clustering process. The primordial sources of the clustering phenomena are thought to be fluctuations in the density of the very early Universe, induced by quantum processes. The clustering process itself is a result of a nonlinear gravitational amplification of these fluctuations [9]. At every level, nonlinear phenomena lead to the emergence of new structures that provide bases for the dynamics of the next higher level. Thus, the matter distribution at large scales is made up of interacting clusters, which, in turn, comprise individual galaxies, which are in fact star distributions and so on.

On scales comparable to stars or galaxy size there have already been several proposals about how the self-organizing systems theory may be applied in astrophysics: the spectra coming from accretion disks suggests that the formation of stars and galaxies is driven by a feedback process specific for stable non equilibrium systems [10].

The evidence we have presently for the large scale organization of the universe comes from different sources: catalogues of galaxy redshifts; absorption lines in quasar spectra; the cosmic background radiation; studies of the distribution of hot ionized gas in clusters of galaxies; measurements of large scale peculiar velocities [11].

In this work we shall show that the observations on galaxy distribution suggest that the large scale structure is the result of a self-organizing phenomenon, related to a two-parameter spatial Verhulst law.

2. CAPACITY DIMENSION

The box-counting method, used here for fractal dimension calculation, is not widely used in galaxy distribution analysis. For this reason we would like to describe it briefly.

We consider a set N_0 of points distributed in a three dimensional Euclidian space. Then we can always cover the set with cubes of edge length equal to ε. Let $N(\varepsilon)$ be the minimum number of cubes needed to cover the set. The capacity dimension is then defined by [13]:

$$D_c = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln \frac{1}{\varepsilon}} \tag{1}$$

In practice, given a distribution of points, we count $N(\varepsilon)$ for different values of edge length and then plot $\ln N(\varepsilon)$ versus $\ln \frac{1}{\varepsilon}$ and find the slope of the linear function which describes this dependence. This slope will be equal to capacity (Hausdorff) dimension.

In the case of a distribution of galaxies it is used the redshift space, *i.e.* the position of each galaxy is given by three numbers: right ascension α, declination δ and redshift z. By transforming the spherical coordinates into Cartesian ones we can apply the box-counting method as it has been described above.

If we compare the correlation function method and the box-counting method then we can see that the second is a purely metric concept, appropriate for geometric characterization. It treats all cubes needed to cover the distribution with no difference. This is a minus of the method, because it implies to construct and check the emptiness of all boxes at each resolution ε and this means additional computing time. But the box-counting method circumvents the problem of assuming a homogeneity cut-off and moreover, there is another advantage: we can use the limit $\varepsilon \to 0$ without fearing that this will affect the results as it is happened, because of the noises, in the limit $r \to 0$ used in correlation function. Another advantage of the box-counting method is that it uses a single set of measured coordinates in calculus, since it takes into account only whether or not a galaxy is contained by a given box.

3. SCALING PROPERTIES OF THE LARGE SCALE STRUCTURES IN 2DFRS: THE VERHULST LAW

The final release of 2dF galaxy redshift survey (14) opens a new possibility for performing different kinds of statistical analysis of large samples of galaxies. The 2dF galaxy redshift survey contains about 222 000 galaxies with measured redshifts in two (NGP and SGP) slices of about 90° × 15° (southern slice) and 75° × 10° (northern slice) complete up to an effective median magnitude $b_j = 19.35$, with the effective median redshift $z_{ef} \approx 0.15$. These data were thoroughly analyzed using the correlation function by Hawkins *et al.* in 2003 [15]. One among their results is that the real-space correlation function, $\xi(r)$ fits well a power-law $(r/r_0)^{-\gamma r}$ on scales $0.1 < r < 12$ h^{-1}Mpc, with the correlation length $r_0 = 5.05 \pm 0.26$ h^{-1}Mpc and $\gamma_r = 1.67 \pm 0.03$. At larger scales, $\xi(r)$ drops below a power-law. Especially this conclusion is very important for this work because it shows that at scales larger than 12 h^{-1}Mpc another approach is needed.

Consider a set of galaxies with angular position on the sky and red shifts. The purpose is to investigate the variation of the fractal dimension with the scale of analysis. The method adopted here is to increase the scale by varying two of the parameters and to calculate, at each step, the fractal dimension of the distribution into the new obtained volume. The parameters to be varied are here the right ascension and the redshift. Thus, starting from a thin slice-shaped volume, we increase its thickness and depth step by step, with 20 Mpc at each step, and compute the fractal dimension each time. In the following, we investigate a sub-sample of galaxies, selected from the Two degree Field Galaxy Redshift Survey (2dFGRS), containing 237233 galaxies with measured redshifts. The sub-sample we have selected contains 32084 objects identified as galaxies, with magnitudes $M < 19.5$, forming a largely contiguous region defined by right ascensions $10^h < \alpha < 14^h$, declinations $-6° < \delta < 2°$ and redshifts $0.05 < \alpha < 0.1076$.

For each step, we have calculated the capacity dimension using the box-counting method as described above. We have counted $N(\varepsilon)$ for twenty different values of edge length and then plot $\ln N(\varepsilon)$ as a function of $\ln(1/\varepsilon)$. The slope of the linear function which describes this dependence is equal to D_c.

To transform the redshifts in distances, in the following it is used the Einstein-de Sitter model ($\Omega_M = 1$, $\Omega_\Lambda = \Omega_k = 0$). Then the distance to a galaxy with redshift z reads

$$r = \frac{c}{H}\int_0^z \frac{dz}{\sqrt{\Omega_M (1+z)^3}} \tag{2}$$

c is here the speed of light and H is the Hubble constant, $H = 70\frac{\text{km}}{\text{s}\cdot\text{Mpc}}$.

Fig. 1 shows the variation of the fractal dimension with the scale.

It is obvious that the fractal dimension increases with the scale and reaches a maximum value at a scale comparable to $L \approx 200$ Mpc while the growth rate is decreasing to zero. The same feature is present in the Verhulst model. Suppose we have r for the growth rate of the fractal dimension (known in Biology as "the Malthusian parameter") and K for the maximum value reached (called "carrying capacity"). Note that the growth rate of the fractal dimension is decreasing to 0 as we increase the scale. The Verhulst equation then reads

$$\frac{dDc}{dL} = \frac{rDc(K - Dc)}{K} \tag{3}$$

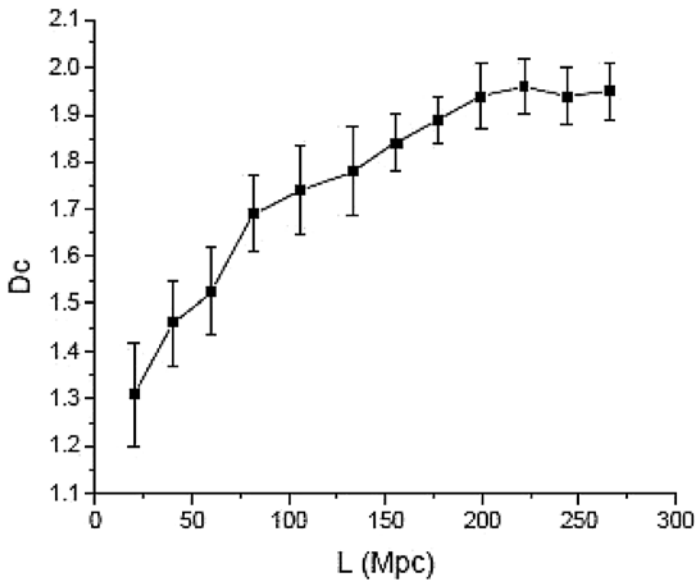

Fig. 1 – The capacity dimension of galaxy distribution as a function of scale.

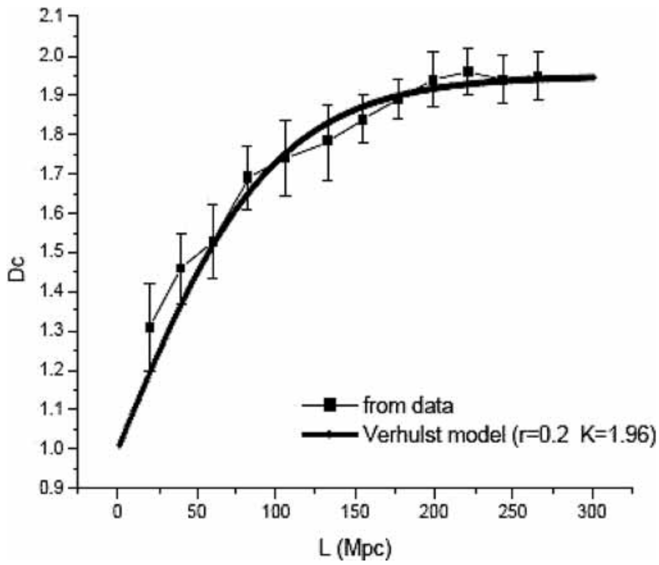

Fig. 2 – Comparison between the data and the Verhulst model.

Using a simple method, we can calculate the parameters r and K [12]. The best fit for our data gives $r = 0.2$ and $K = 1.96$. Thus, the Verhulst equation is

$$\frac{dDc}{dL} = 0.2Dc\left(1 - \frac{Dc}{1.96}\right) \tag{4}$$

4. CONCLUDING REMARKS

In this paper we have presented results from an analysis of the fractal dimension of galaxy distribution at different scales, performed for a sample selected from the Two degree Field Galaxy Redshift Survey. As we expected, the fractal dimension is growing with the scale and reaches a maximum value equal to two at a scale comparable to 200 Mpc. It was shown that the logistic equation approach captures the overall scaling behavior and additionally it provides good estimates of the parameters. Since the Verhulst law is specific for the nonlinear growth phenomena, this result is in perfectly concordance with the nonlinear theory of structure formation and argues for a self-organized universe.

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