APPLICATION OF THE WIENER PREDICTIVE FILTER AND THE BENIOFF GRAPHS TO FORECAST VRANCEA (ROMANIA) EARTHQUAKES

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In this article I continue the forecasting study of Vrancea earthquakes based on the extrapolation in time of the square root of the seismic energy E_s. For this purpose, I use two alternative approaches based on the theory of Wieners filters and Benioff graphs, respectively. The application of the first method indicates that the next major Vrancea earthquake ($M_{GR} \geq 6.5(6.7)$) will occur in the time period 2016–2020, while according to the second method the major event would occur earlier, in the time period 2011–2015.

Key words: time series, seismic energy, Wiener filters, Benioff graphs, earthquake prediction.

1. INTRODUCTION

A first attempt to apply the Wiener filter to Vrancea (Romania) earthquakes was done in 1975 [1]. In a recent work [2], the temporal evolution of the state of stress in the Vrancea region was estimated using Benioff graphs [3] and data from a previous work [4]. In this study I continue the previous forecasting research using updated earthquake data.

It is considered that part of the elastic energy E, which is stored up during the earthquake preparation stage, changes into seismic energy E_s. According to Benioff [3], the square root of the energy E_s is proportional to the elastic force that generates the earthquake. Based on these physical considerations, I decided to use the square root of E_s as a reliable estimator of the state of stress in the Vrancea region and a possibly reliable predictive parameter. The computation of the seismic energy E_s is done using the procedure described in [2].

2. METHODS

2.1. WIENER FILTERS

The input for the Wiener filter [5] is represented by the discrete values of the parameter that characterizes the past seismic activity in the Vrancea intermediate-depth seismic region. The input time series can be written as: X_{t-1}, X_{t-2}, … X_{t-n}. As explained in the *Introduction* the seismic parameter to be used in this paper is the square root of the seismic energy, E_s.

We define the Wiener type operator W_k, with predictive distance m, which is characterized by its coefficients W_1, W_2, W_3 …, W_n. The output of the filter at time $t + m$, is calculated as the discrete convolution between the time series X_{t-k} and the predictive operator W_k:

$$X'_{t+m} = W_1 X_{t-1} + W_2 X_{t-2} + \ldots W_n X_{t-n} = \sum_{k=1}^{n} W_k X_{t-k}. \tag{1}$$

The convolution expressed by relation (1) represents an estimation of the parameter which characterizes the seismic activity in the Vrancea region at time $t + m$ $(m = 0, 1, 2\ldots)$. Obviously, this estimation contains some errors: calculation errors, errors caused by the deviation of the time series from stationarity, etc.

The true value which will characterize the seismic activity in Vrancea at the time $t + m$ is X_{t+m}, and it represents the desired output of the predictive filter. Thus, the difference:

$$\epsilon_{m,n} = X_{t+m} - X'_{t+m} = X_{t+m} - \sum\nolimits_{k=1}^{n} W_k X_{t-k} \tag{2}$$

represents the error of our estimated value for the time $t + m$.

The main problem in designing our filter is the determination of the W_k coefficients. To determine W_k we minimize the mean square error between the desired output and the actual one:

$$\sigma^2_{m,n} = M \left| X_{t+m} - \sum_{k=1}^{n} W_k X_{t-k} \right| \to \min. \tag{3}$$

To minimize $\sigma_{m,n}$, it is required that:

$$\frac{\partial \sigma^2_{m,n}}{\partial W_k} \equiv \sum_{l=1}^{n} W \phi_{xx}(l-k) - \phi_{xx}(k+m) = 0; \quad k = 1, 2, \ldots, n. \tag{4}$$

where $\Phi_{xx}(\tau)$, $\tau = 0$, $k + m$, $l - k$ represents the auto-correlation function of the time series at the Wiener filter input.

The computation of the auto-correlation functions $\Phi_{xx}(\tau)$ and the solution of the system of equations for various values of m lead to the estimation of the coefficients W_k. Using relation (1), we can estimate the value of the seismicity parameter X'_{t+m} for the next period $t + m$.

2.2. BENIOFF GRAPHS

Plotting a Benioff graph actually adds up to determining the energy that a seismic focus will release in the form of elastic waves over a period of time the length of which is chosen according to one's purpose. What matters, as we plot the Benioff graph, is that the seismic efficiency *e*, defined as the ratio between the seismic energy, E_s, and the elastic energy, *E*, is always the same for a given seismic zone.

3. RESULTS

3.1. WIENER FILTER APPROACH

The applied procedure is based on the statistical correlations that exist between the known values of $E_s^{1/2}$ of the past and the ones we would like to predict by extrapolation to the future.

The calculation of the auto-correlation functions, the solution of the system of equations to estimate the future values $\sum E_{s,i}^{1/2}$, as well as the computation of the errors σ were done with the aid of the formulae:

$$\sigma = \left[\Phi_{xx}(0) - \sum_{k=1}^{n} W_k \Phi_{xx}(k+m) \right]^{1/2} \tag{5}$$

The input time series was obtained by calculating the square root of the seismic energy of Vrancea intermediate-depth earthquakes for the time periods 1936–1940, 1941–1945,…., 1995–2000 and 2001–2005. From the data, we get the following values for the parameter $X_{t-k} = \left(\sum_{i=1}^{p_k} E_{s,i}^{0.5} \right)_{t-k}$: 54.913, 25.787, 15.738, 7.822, 7.819, 5.344, and 4.688, for the time periods 1936–1940, up to 1966–1970, respectively. The parameter p_k represents the number of earthquakes that were considered in the time interval $t - k$. The time periods 1971–1975 up to 2001–2005 are characterized by the following values X_{t-k}: 2.111, 24.817, 7.539, 28.755, 4.638, 6.332 and 8.500, respectively.

The predicted data (prediction interval $p = 4$) for the time periods 1971–1975, 1976–1980, …, 1996–2000, 2001–2005 is represented by the values 2.363, 24.012, 13.417, 28.755, 8.632, 4.069 and 6.437, respectively. For the time periods 2006–2010, 2011–2015 2016–2020 and 2021–2025 the predicted values are 4.968, 3.330, 11.256 and 4.247.

Considering the same time intervals, we have also used the centered values $X_{t-k} = \left(\sum_{i=1}^{p_k} E_{s,i}^{0.5}\right)_{t-k} - \overline{X}$, where p_k is the number of earthquakes considered from the interval $t-k$ and $\overline{X}$ is the mean of the values $\left(\sum_{i=1}^{p_k} E_{s,i}^{0.5}\right)_{t-k}$ for the entire sequence of earthquakes used, namely: $\overline{X} = M\left|\left(\sum_{i=1}^{p_k} E_{s,i}^{0.5}\right)_{t-k} = \right|$ $= \frac{1}{n}\left[\sum_{i=1}^{n}\left(\sum_{i=1}^{p_k} E_{s,i}^{0.5}\right)_{t-k}\right]$. The centered values X_{t-k} for the time periods 1936–1940 up to 1966–1970 are 44.206, 15.080, 5.031, –2.884, –2.887, –5.362 and –6.015, respectively. The periods 1971–1975 up to 2001–2005 are characterized by the following centered values X_{t-k}: –8.595, 14.110, –3.167, 18.048, –6.068, –4.374 and –2.206, respectively. The average value $\overline{X}$ = = 10.706.

The predicted data (prediction interval $p = 4$) for the time periods 1971–1975, 1976–1980, …., 1996–2000 and 2001–2005 is represented by the values 6.672, 18.839, 12.796, 21.117, 9.081, 6.559 and 8,426, respectively, and for the time periods 2006–2010, 2011–2015, 2016–2020 and 2021–2025 by the values 6.438, 7.889, 11.986 and 6.527, respectively.

The above calculations are synthesized in Table 1. The predictions made for a prediction interval $p = 4$ are similar with those obtained for $p = 2$ or $p = 3$. From these data it is noticed that the time period with a maximum predicted value of the parameter $\sum_{i=1}^{p_k} E_{s,i}^{0.5}$ is 2016–2020, for both absolute-values and centered-values cases.

Table 1

Sampling interval $\Delta t = 5$ years

Type of series	Time interval Δt (years)	Value $\sum E_{s,i}^{1/2} \pm \sigma$ predicted (10^{10}erg$^{1/2}$)
Actual series Absolute values	2006–2010	4.967 ± 2.24
	2011–2015	3.330 ± 2.16
	2016–2020	11.255 ± 2.32
	2021–2025	4.247 ± 2.43
Actual series Centered values	2006–2010	6.439 ± 2.18
	2011–2015	7.889 ± 2.29
	2016–2020	11.987 ± 2.16
	2021–2025	6.528 ± 2.08

3.2. BENIOFF GRAPHS APPROACH

Using a similar approach as that in [2], *I* calculated the seismic energy E_s for Vrancea earthquakes from 1796 to 2005. In [2] we considered that the time interval 1796–1889 and especially previous time intervals (for example, 1500–1796 or 1000–1796) could be deficitary. It was also shown that by considering the time interval 1890–2005, we have a fairly complete data set.

The stepwise graph of Fig. 1 shows the cumulative value $\sum E_{s,i}^{1/2}$ *versus* time. Also appearing on the time axis are the 36-to-38-year cycles first singled out in [6] and then corrected and reused in [4] for Vrancea earthquake prediction. It should be remembered that a half cycle is characterized by earthquakes of magnitudes $M_{GR} < 6.5$ (6.7), while earthquakes of $M_{GR} \geq 6.5(6.7)$ are typical of the other half. As expected, the average slopes of the Benioff graph for every half-cycle with $M_{GR} < 6.5$ (6.7) is less steep than that for its counterpart characterized by $M_{GR} \geq 6.5$ (6.7). In Fig. 1 we found the segments α and γ to be parallel to each other. The graph may also suggest that for the periods of time prior to 1935, the year when a seismic network began to be established in Romania, the data could be incomplete or the magnitudes underrated.

For the extrapolation in Fig. 1, we assume that the slope of the Benioff graph for the half-cycle that starts after 2005 should be similar with the slope of the β segment or *Z* segment that both characterize half cycles with high seismic energy release.

Fig. 2 shows the same Benioff graph as that in Fig. 1. However, the segments ρ, η and δ of Fig. 2 characterize full cycles (of 36–38 years). The extrapolation of the δ segment in Fig. 2 is done based on the assumption that the segments ρ and δ should have similar slopes. We note in both Figs. 1 and 2 the equality of the segments *AB* and *BC*.

It is noticed that the hypothesis made in Fig. 1, as well as that made in Fig. 2, suggest that the next major Vrancea earthquake ($M_{GR} \geq 6.5(6.7)$) will occur in the 2011–2015 time window.

4. DISCUSSION AND CONCLUSIONS

In the 1996 paper (D. Enescu and B. D. Enescu, [4]) we have shown that almost certainly the next major Vrancea earthquakes ($M_{GR} \geq 6.5(6.7)$) will not occur before the end of the year 2005. We hypothesized that the next major event will occur in one of the years 2006, 2007 or 2008 [4].

In [4] we made also a different hypothesis that the next $M_{GR} \geq 6.5(6.7)$ earthquake could occur later than the year 2010. The predictions based on the

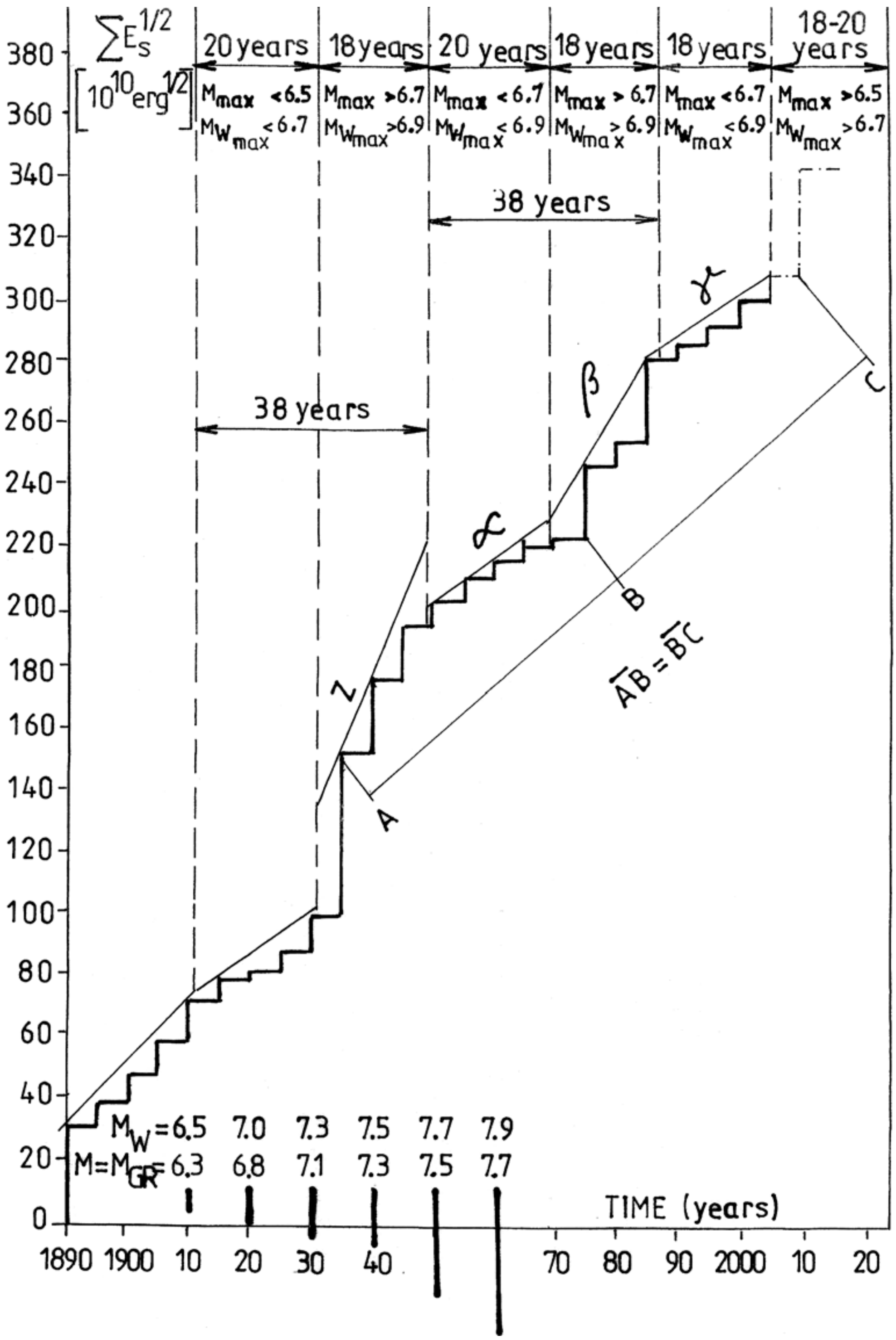

Fig. 1 – Benioff graph for Vrancea earthquakes occurred in the period 1890–2005, with average slope for every half cycle.

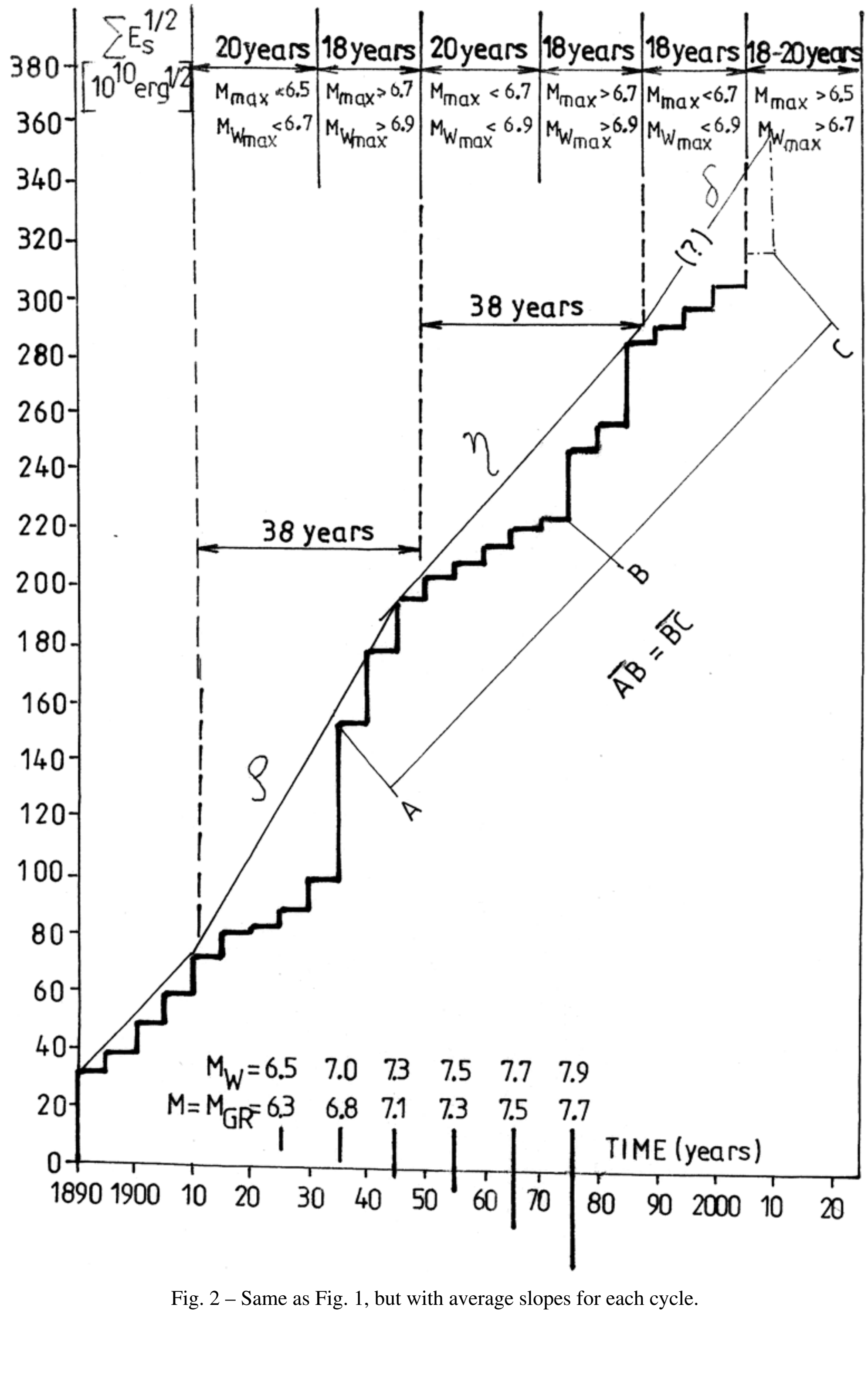

Fig. 2 – Same as Fig. 1, but with average slopes for each cycle.

"Wiener filter" and "Benioff graph" methods seem to support this second hypothesis.

We can summarize the conclusions of the present paper as follows:

1) The results obtained using the predictive Wiener filter suggest that the next major Vrancea earthquake ($M_{GR} \geq 6.5(6.7)$) will occur in the time period 2016–2020.
2) However, the Benioff graphs seem to indicate that the next major Vrancea event will occur earlier, in the time period 2011–2015. Further study is necessary to understand the differences between the two forecasts.
3) The Vrancea earthquakes with M < 6.5 could occur any time; they are not considered dangerous.

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