THE EFFECT OF SLIP CONDITION ON UNSTEADY MHD OSCILLATORY FLOW OF A VISCOUS FLUID IN A PLANER CHANNEL

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Oscillatory motion of a magnetohydrodynamic fluid with heat transfer analysis through a porous planar channel filled with a saturated porous medium is considered. It is assumed that the no-slip condition between the wall and the fluid remains no longer valid. The effect of the wall slip on velocity field is studied. Also the results are discussed through graphs.

Key words: MHD; heat transfer; radiation effect; porous channel; slip condition.

1. INTRODUCTION

At the macroscopic level it is well accepted that the boundary condition for a viscous fluid at a solid wall is one of “no-slip”, *i.e.*, the fluid velocity matches the velocity of the solid boundary. While the no-slip boundary condition has been proven experimentally to be accurate for a number of macroscopic flows, it remains on assumption that is not based on physical principles. In fact, nearly two hundred years ago Navier proposed a general boundary condition that incorporates the possibility of fluid slip at a solid boundary. Navier’s proposed condition assumes that the velocity, v_x, at a solid surface is proportional to the shear stress at the surface [1, 2]

$$v_x = \gamma\left(dv_x/dy\right)$$

where γ is the slip strength or slip coefficient. If $\gamma = 0$ then the general assumed no-slip boundary condition is obtained. If γ is finite, fluid slip occurs at the wall but its effect depends upon the length scale of the flow. The above relation states that the velocity of the fluid at the plates is linearly proportional to the shear stress at the plate. Also, one could impose nonlinear slip boundary conditions (*e.g.* [3]).

The fluid slippage phenomenon at the solid boundaries appear in many applications such as in microchannels or nanochannels and in applications where

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a thin film of light oil is attached to the moving plates or when the surface is coated with special coatings such as thick monolayer of hydrophobic octadecyltrichlorosilane [4]. Also, wall slip can occur in the working fluid contains concentrated suspensions [5]. However, the literature lacks studies that take into account the possibility of fluid slippage at the wall under vibrating conditions. Recently, several researchers have suggested that the no-slip boundary condition may not be suitable for hydrophilic flows over hydrophobic boundaries at both the micro and nano scale (for detailed study the reader is referred to see [6–8]). The effect of the fluid slippage at the wall for coutte flow are considered by Marques *et al.* [9] under steady state conditions and only for gases. The closed form solution for steady periodic and transient velocity field under slip condition have been studied by Khaled and Vafai [10]. The effect of slip condition on MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi [11].

In this paper we have extended the work of Makinde and Mhone [12] by considering the fluid slip at the lower wall. Our primary objective is to study the slip effects on the velocity field and the effect of other parameters present in equations on the fluid slip. Exact analytic solution is presented. It is noted that our present solution reduces to the Makinde and Mhone results by taking the slip parameter equal to zero which provides a useful mathematical check. The paper is organized into three sections. Section 2 consists of the flow analysis which contains the governing equations, their solution, graphical representation of results and their discussion. Section 3 contains come concluding remarks.

2. FLOW ANALYSIS

We consider an incompressible, viscous and electrically conducting fluid bounded by two parallel plates separated by a distance a [12]. The channel is assumed to be filled with a saturated porous medium. A uniform magnetic field of strength B_0 is applied perpendicular to the plates. The above plate is heated at constant temperature and the radiation effect is also taken in to account. Therefore, the governing equations for this flow geometry in dimensionless form are given by [12]:

$$\mathrm{Re}\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-\left(s^2+H^2\right)u+Gr\theta, \tag{1}$$

$$Pe\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}-N^2\theta, \tag{2}$$

subject to the boundary conditions

$$u-\gamma\frac{\partial u}{\partial y}=0,\quad \theta=0,\quad \text{at}\quad y=0,3$$
$$u=0,\quad \theta=1,\quad \text{at}\quad y=1. \tag{3}$$

where Re is the Reynold's number, Gr is the Grashoff number, H is the Hartmann number, N is the radiation parameter, Pe is the Peclet number, s is the porous medium shape factor parameter, and γ is the dimensionless slip parameter. For purely an oscillatory flow we take the pressure gradient of the form $-\frac{\partial p}{\partial x}=\lambda e^{i\omega t}$ where λ is a constant and ω is the frequency of oscillations. Due to the selected form of pressure gradient we assume the solution of eqs. (1), (2) of the form

$$u(y,t)=u_0(y)e^{i\omega t},\quad \theta(y,t)=\theta_0(y)e^{i\omega t}. \tag{4}$$

By substituting eq. (4) in (1)–(3), we have

$$u_0''-m_2^2u_0=-\lambda+Gr\,\theta_0, \tag{5}$$

$$\theta_0''+m_1^2\theta_0=0, \tag{6}$$

with boundary conditions

$$u_0-\gamma u_0'=0,\quad \theta_0=0,\quad \text{at}\quad y=0,$$
$$u_0=0,\quad \theta_0=1,\quad \text{at}\quad y=1, \tag{7}$$

where $'$ denotes the differentiation with respect to y, $m_1=\sqrt{N^2-i\omega Pe}$ and $m_2=\sqrt{s^2+H^2+i\omega Re}$.

Equation (6) admits the solution

$$\theta_0(y)=\frac{\sin m_1y}{\sin m_1}. \tag{8}$$

Substituting Eq. (8) in eq. (5) and then solving it, we get

$$u_0(y)=C_1\cosh m_2y+C_2\sinh m_2y+\frac{\lambda}{m_2^2}+\frac{Gr}{\sin m_1(m_1^2-m_2^2)}\sin m_1y, \tag{9}$$

where C_1 and C_2 are defined through

$$C_1=-\frac{\lambda}{m_2^2}+\gamma\left(C_2m_2+\frac{Grm_1}{\sin m_1(m_1^2-m_2^2)}\right), \tag{10}$$

and

$$C_2 = \frac{1}{\sinh m_2 + \gamma m_2 \cosh m_2}\left[\left(\cosh m_2 - 1\right)\frac{\lambda}{m_2^2} - \right. \\ \left. - \frac{Gr}{(m_1^2 - m_2^2)}\left(1 + \frac{\gamma m_1 \cosh m_2}{\sin m_1}\right)\right]. \tag{11}$$

The expression for the shear stress is given by

$$\tau(y,t) = \left(C_1 m_2 \sinh m_2 y + C_2 m_2 \cosh m_2 y + \frac{Gr m_1}{\sin m_1 (m_1^2 - m_2^2)}\cos m_1 y\right)e^{i\omega t}. \tag{12}$$

2.1. GRAPHICAL RESULTS AND DISCUSSION

In order to see the physical impact of the slip at the wall on velocity field, the graphical representation of results is important. In figs. 1–7 we have plotted the velocity $u(y,t)$ for different values of the parameters present in solution

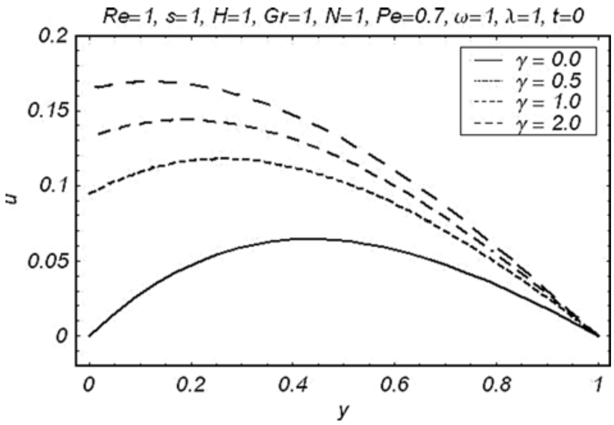

Fig. 1 – Effect of wall slip on the velocity u.

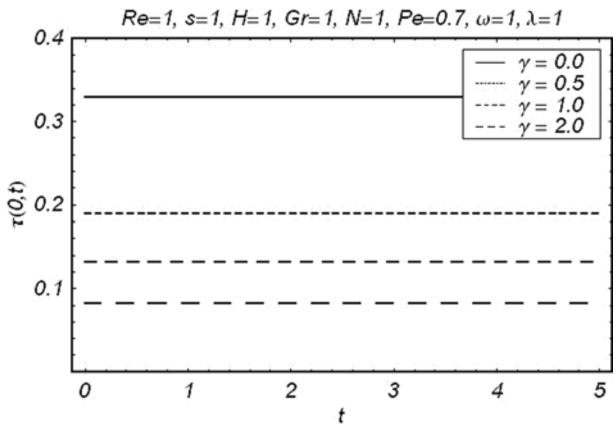

Fig. 2 – Effect of wall slip on shear stress at the plate.

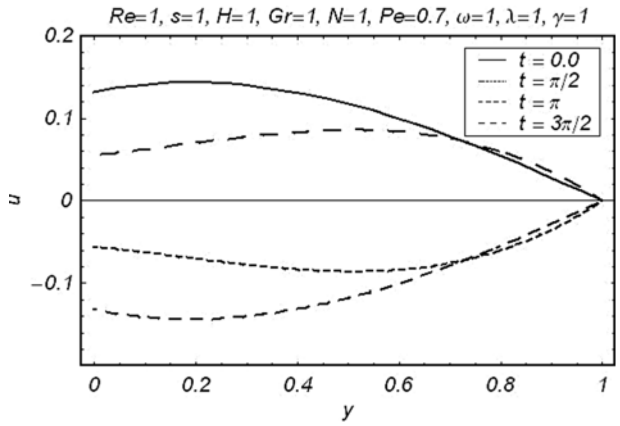

Fig. 3 – Velocity profile at different times.

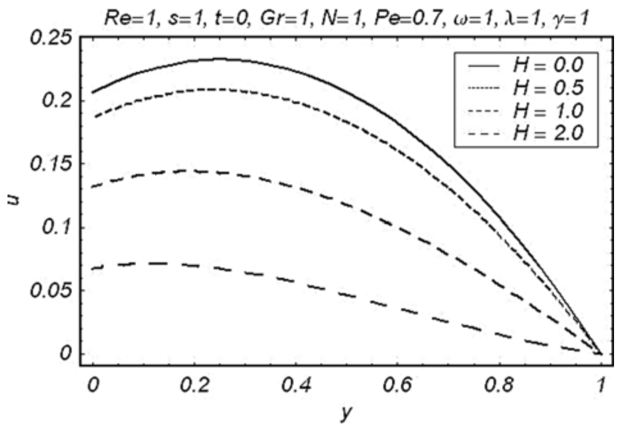

Fig. 4 – Velocity curves for different values of the Hartmann number H.

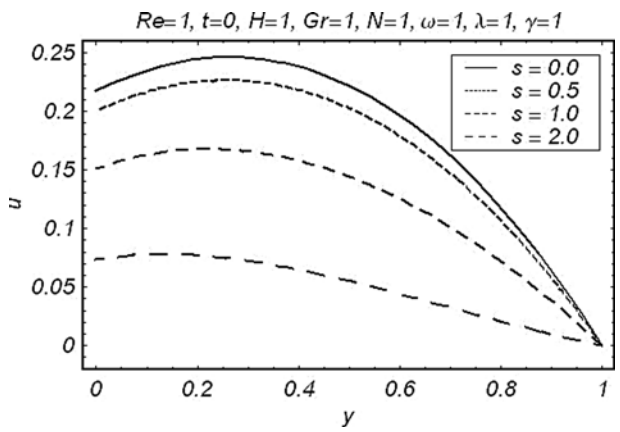

Fig. 5 – The velocity u for different values of the porosity parameter s.

expressions. In Fig. 1 we have plotted the velocity $u(y,t)$ against y for different values of the slip parameter γ. Clearly, by increasing the slip at the wall the velocity increases at the wall whereas the effect is totally reversed in the case of

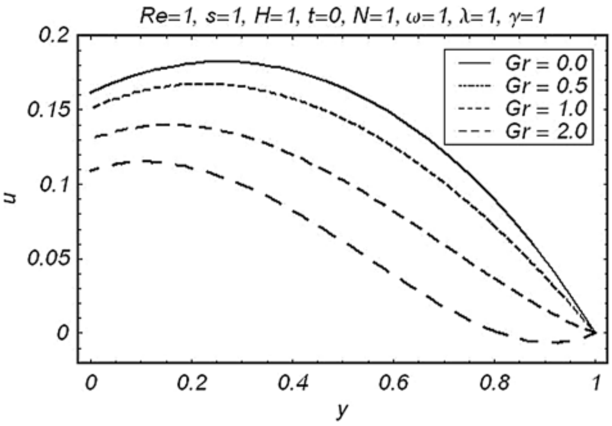

Fig. 6 – Velocity behavior at different values of the Grashoff number *G*.

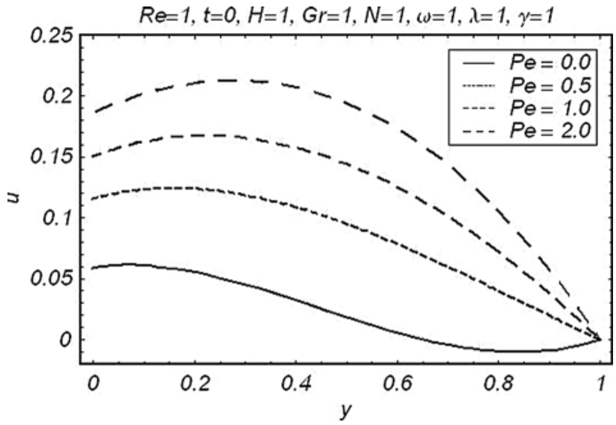

Fig. 7 – Velocity curves for different values of the Peclet number *Pe*.

shear stress at the wall which is in accordance to the existing results in literature. Fig. 3 shows that the velocity oscillates in equal intervals of time and the effect of the slip condition can clearly be seen at the wall. It is worth mentioning here that the strength of slip at the wall is affected by the different parameters involved in the equations. Fig. 4 is plotted for different values of the Hartmann number H at fixed value of the slip parameter γ. It is clear from the graph that the strength of the slip at the wall decreases by increasing H. Similar effects of the porosity parameter s and the Grashoff number Gr are observed in Figs. 5 and 6 respectively. However, by increasing Peclet number Pe the velocity at the wall also increases which shows that the Peclet number causes to strengthen the fluid slip at the wall as shown in Fig. 7.

3. CONCLUDING REMARKS

In this paper unsteady oscillatory flow of an incompressible viscous fluid in a planer channel filled with porous medium in the presence of a transverse magnetic field is studied. The effect of fluid slip at the lower wall is to increase the velocity at the wall. Further it is observed that the Hartmann number, the porosity parameter and the Grashoff number cause to weaken the slip at the wall whereas the effect of the Peclet number is to strengthen the slip.

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